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IFAC-PapersOnLine 48-22 (2015) 001–007

A deterministic gossiping algorithm for the synchronization of multi-agent systems ⋆

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Abstract: The main objective of the synchronization of multi-agent systems is the asymptotic convergence of the agents towards a common trajectory. This paper considers the synchronization of linear discrete-time agents over a network in which only point-to-point connections are allowed. The main result is a new control algorithm designed to ensure asymptotic synchronization of the networked agents. The control algorithm of each agent works in the following steps: (i) establish a coupling to a neighboring agent that is currently not connected to another agent. (ii) synchronize the coupled agents in finite time and search for another agent to be coupled. A deterministic coupling of the agents is required and achieved by the use of a periodically changing coupling sequence. Considering the synchronization of integrators reveals the similarity to a gossiping algorithm. However, this paper extends the results reported in the literature because a necessary and sufficient condition for the synchronization of discrete-time multi-agent systems with arbitrary linear dynamics is presented.

Keywords: synchronization, gossiping, multi-agent systems, dead-beat control

1. INTRODUCTION

The achievement of a coherent behavior of a large number of autonomous systems (agents) is a typical control objective in networked systems. Examples can be found in the synchronization of sensors or autonomous mobile agents which are able to exchange information over a wireless communication network.

This paper considers the synchronization of autonomous discrete-time agents. The requirement of asymptotic synchronization

$$\lim_{k\to\infty} \|\boldsymbol{x}_i(k) - \boldsymbol{x}_j(k)\| = 0, \quad i, j = 1, 2, \ldots, N \qquad (1)$$

of the agent states $\boldsymbol{x}_i(k)$ represents a common control objective which can only be reached if couplings among the overall system are introduced through which the input $u_i(k)$ of every agent is influenced by every other agent directly or indirectly. Therefore, the synchronizing controller has to be chosen to create the required couplings. In this paper the networked controller consists of two components, the local control algorithms R_i, $i = 1, 2, \ldots, N$ and the communication network (Fig. 1).

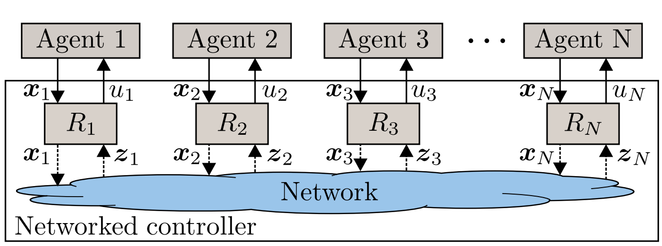

Fig. 1. Networked multi-agent system

Typically, synchronization is considered for static or randomly changing network structures. Besides these types of networks there also exist wireless ad hoc networks, where information can only be exchanged by point-to-point connections. This paper is motivated by such networks, where it is assumed that each agent can only exchange information with one other agent at a time. The main contribution of this paper is a new control algorithm which ensures asymptotic synchronization (1) of the networked agents. The control algorithm R_i of the i-th agent works in two basic steps:

1. Establish a coupling to a neighboring agent that is currently not coupled with another agent.
2. Synchronize with the coupled agent in finite time and go back to step 1.

Depending on the structure of the network, not every agent can be coupled to every other agent. The synchronization of a coupled pair of agents in finite time is achieved by the use of a dead-beat controller. It is shown that a coupled pair of agents reaches synchronization in at most n time steps, where n is the dynamical order of the agents.

In literature, this kind of synchronization procedure is known as gossiping. Typically, gossiping algorithms are used for the distributed calculation of the averaged value of integrator states (e.g. in sensor networks). This paper extends the existing literature to discrete-time multi-agent systems with arbitrary linear dynamics.

The control algorithms R_i are required to guarantee a deterministic behavior of the multi-agent system that is achieved by the use of a periodically changing sequence of the agent couplings. For the periodically changing interconnection behavior, a synchronization condition is derived and it is shown that asymptotic synchronization of unstable agents is achieved if and only if the synchroniza-

⋆ This work is supported by the German Research Foundation (DFG) under grant LU 462/31-1.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/ifacol.2015.10.298

tion of the coupled pairs of agent happens faster than the agent's divergence towards their unstable behavior.

Literature survey. The existing literature on synchronization has focused almost exclusively on the design of synchronizing controllers for static and dynamic networks or for nonidentical systems. Most of the literature deals with the synchronization of multi-agent systems based on a continuous information exchange (e.g. Olfati-Saber et al. (2007); Li et al. (2010); Mosebach and Lunze (2014); Mosebach et al. (2015); Hengster-Movric et al. (2015)). However, there is an increasing amount of literature considering time-varying networks. In Scardovi and Sepulchre (2009) a controller for the synchronization of uniformly connected and neutrally stable agents is designed. Synchronization of unstable agents in time-varying networks was considered in Yu et al. (2012), Kim et al. (2013) and Qin and Yu (2014). The design methods in all these publications are motivated by real networks, where robustness against link failures or packet losses is of interest.

In contrast to the mentioned literature this paper is motivated by synchronization of multi-agent systems connected over peer-to-peer or ad hoc networks in which only a pairwise information exchange between agents is allowed. Hence, the networked controller is designed to change the coupling of an agent pair whenever synchronization of the coupled agents is achieved. This particular type of algorithms, known as gossip algorithms, has received increasing attention in the literature. It is useful to distinguish between algorithms with a probabilistic agent coupling given in Boyd et al. (2006) or a deterministic one in Liu et al. (2011). In Mou et al. (2010) it is shown that a periodic information exchange between agents can be used to achieve a deterministic behavior like it is done in the present paper. A self-triggered gossiping algorithm, where each agent independently determines the time instants for an information exchange, is presented in De Persis et al. (2013). Recent publications investigate the improvement of the convergence rate of the algorithms (Liu et al. (2013); Mangoubi et al. (2013)).

However, gossiping algorithms are used for the distributed averaging of scalar values, which is similar to the asymptotic synchronization of simple integrator dynamics. This paper extends the existing literature to the synchronization of unstable discrete-time agents with arbitrary linear dynamics.

Structure of this paper. In Section 2, the notation, some results from graph theory and properties of stochastic matrices are given. The agent model and important assumptions are given in Section 3. The main result is the control algorithm derived in Section 4. First, the structure and the operation of the control algorithms R_i are explained. Secondly, a controller design method is presented, the behavior of the overall closed-loop system is analyzed and a necessary and sufficient synchronization condition is derived. Section 5 illustrates the control algorithm by its application to the synchronization of harmonic oscillators.

2. PRELIMINARIES AND NOTATION

2.1 Notation

Scalars are represented by italic letters (a), vectors and matrices by bold face letters ($\boldsymbol{x}$, $\boldsymbol{A}$). The identity matrix is symbolized by $\boldsymbol{I}$. Sets are denoted by calligraphic letters ($\mathcal{P}$). The i-th eigenvalue of $\boldsymbol{A}$ is denoted by $\lambda_i(\boldsymbol{A})$ and its spectral radius by $\rho(\boldsymbol{A}) = \max_i(|\lambda_i(\boldsymbol{A})|)$. The i-th entry of a vector $\boldsymbol{x}$ is given by $(\boldsymbol{x})_i$. The set of real numbers and the set of integers are represented by $\mathbb{R}$ and $\mathbb{Z}$, respectively. $\Re\{z\}$ denotes the real part of a complex number z. The Kronecker product of two matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ will be expressed by $\boldsymbol{A} \otimes \boldsymbol{B}$. $\mathbb{1} = (1 \dots 1)^T$ is the one vector, $\boldsymbol{0} = (0 \dots 0)^T$ the zero vector. $\lceil x \rceil$ is used for the representation of the ceiling function, which is defined by $\lceil x \rceil = \min\{n \in \mathbb{Z} \,|\, n \geq x\}$.

2.2 Graph theory and doubly stochastic matrices

In the following an undirected graph is used for the representation of the network structure. An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a vertex set $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ and an edge set $\mathcal{E} = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \dots, \boldsymbol{e}_M\}$, where self-loops are forbidden. In this paper, the vertex $v_i \in \mathcal{V}$ of the graph $\mathcal{G}$ is associated with the i-th agent and an edge $\boldsymbol{e}_k = \{v_i, v_j\} \in \mathcal{E}$ with a coupling between the i-th and the j-th agent. If $\{v_i, v_j\} \in \mathcal{E}$, then agent j is referred to as a *neighbor* of agent i. The set of neighbors of agent i is defined by $\mathcal{N}_i = \{v_j \,|\, \{v_i, v_j\} \in \mathcal{E}\}$. The Laplacian matrix of the graph $\mathcal{G}$ is given by

$$\boldsymbol{L} = \sum_{k=1}^{M} \epsilon_k \epsilon_k^T, \tag{2}$$

where $(\epsilon_k)_i = 1$ and $(\epsilon_k)_j = -1$ for every $\boldsymbol{e}_k = \{v_i, v_j\} \in \mathcal{E}$. The graph $\mathcal{G}$ is referred to as *connected* if $\lambda_2(\boldsymbol{L}) > 0$, where $\lambda_N(\boldsymbol{L}) \geq \lambda_{N-1}(\boldsymbol{L}) \geq \dots \geq \lambda_1(\boldsymbol{L})$ holds. Note, that every Laplacian matrix has a zero eigenvalue ($\lambda_1 = 0$) with the eigenvector $\boldsymbol{v}_1 = \mathbb{1}$: $\boldsymbol{L}\mathbb{1} = \boldsymbol{0}$.

If every agent is allowed to be coupled to only one other agent, it is clear that the corresponding Laplacian matrix given by (2) fulfills

$$\epsilon_i^T \epsilon_j = 0 \;\; \forall \;\; i \neq j, \;\; i, j = 1, 2, \dots, M. \tag{3}$$

An example for the interconnection of pairwise coupled agents is given in Fig. 3 with the corresponding Laplacians in Fig. 5. Considering $\mathcal{L} = \{\boldsymbol{L}_1, \boldsymbol{L}_2, \dots, \boldsymbol{L}_W\}$ as the set of Laplacian matrices (2) satisfying the property (3), it is easy to verify that

$$\boldsymbol{L}_i^2 = 2\,\boldsymbol{L}_i \;\; \forall \;\; \boldsymbol{L}_i \in \mathcal{L} \tag{4}$$

holds. The structure of a graph, which is described by the Laplacians $\boldsymbol{L}_i \in \mathcal{L}$ can also be described by the non-negative matrices

$$\boldsymbol{T}_i = \boldsymbol{I} - \frac{1}{2}\boldsymbol{L}_i, \quad \boldsymbol{L}_i \in \mathcal{L}, \tag{5}$$

which have some interesting properties. The properties $\boldsymbol{T}_i\mathbb{1} = \mathbb{1}$ and $\mathbb{1}^T\boldsymbol{T}_i = \mathbb{1}^T$ of the non-negative matrices $\boldsymbol{T}_i$ show that these are doubly stochastic and hence have at least one eigenvalue $\lambda_1 = 1$ with the eigenvector $\boldsymbol{v}_1 = \mathbb{1}$. From (4) and (5) it is easy to see that the following holds:

$$\boldsymbol{T}_i^n = \boldsymbol{T}_i \quad \text{and} \quad \boldsymbol{T}_i\boldsymbol{L}_i = \boldsymbol{O}. \tag{6}$$

The matrix product $\boldsymbol{T} = \boldsymbol{T}_P \cdots \boldsymbol{T}_2\,\boldsymbol{T}_1$, $P \in \mathbb{N}$ which is often used in the literature on periodic gossiping appears to be important for the synchronization analysis considered in this paper. An important result in the literature is that

$$\lim_{k \to \infty} \boldsymbol{T}^k = \frac{1}{N}\mathbb{1}\mathbb{1}^T \tag{7}$$

if and only if the corresponding graph $\mathcal{G}$ described by the Laplacian matrix $\boldsymbol{L} = \boldsymbol{L}_1 + \boldsymbol{L}_2 + \dots + \boldsymbol{L}_P$ is connected. For a proof, see, e.g., Liu et al. (2011).