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The Journal of China Universities of Posts and Telecommunications

February 2017, 24(1): 77–86
www.sciencedirect.com/science/journal/10058885

http://jcupt.bupt.edu.cn

Modeling and reachability analysis of synchronizing transitions bounded Petri net systems based upon semi-tensor product of matrices

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Abstract

The reachability problem of synchronizing transitions bounded Petri net systems (BPNSs) is investigated in this paper by constructing a mathematical model for dynamics of BPNS. Using the semi-tensor product (STP) of matrices, the dynamics of BPNSs, which can be viewed as a combination of several small bounded subnets via synchronizing transitions, are described by an algebraic equation. When the algebraic form for its dynamics is established, we can present a necessary and sufficient condition for the reachability between any marking (or state) and initial marking. Also, we give a corresponding algorithm to calculate all of the transition paths between initial marking and any target marking. Finally, an example is shown to illustrate proposed results. The key advantage of our approach, in which the set of reachable markings of BPNSs can be expressed by the set of reachable markings of subnets such that the big reachability set of BPNSs do not need generate, is partly avoid the state explosion problem of Petri nets (PNs).

Keywords reachability, Petri nets, BPNSs, semi-tensor product (STP) of matrices, synchronizing transitions

1 Introduction

PNs are a graph-based mathematical formalism for the design and analysis of discrete event dynamic systems (DEDSs) [1–4] and hybrid system [5–6]. The property of reachability for the PNs that is essential to other dynamic properties [7–8], was studied in a lot of Refs. [9–11]. It is well known that the state equation [7,12–13] and reachability graph (RG) [14–15] are two commonly approaches for reachability analysis. The former is used to test marking non-reachability, the latter is applicable for BPNSs to generate all reachable markings. For BPNSs, the RG approach is inevitably suffer from state explosion problem, although it is a convenient approach which contains full information for the dynamics of BPNSs. There is a wide literature on the state explosion problem, due to the importance in the PNs [16–19]. However, most of the existing literature has not deal with this problem by an algebraic approach. The STP of matrices that was initially proposed by Cheng et al. [20–21] makes it possible to partly avoid the state problem by solving reachability problem in an algebraic form.

The recent study [22–23] have been proven that PNs can be studied by the STP of matrices, due to the fact that the PNs can be converted into a linear system under the framework of it, which has also been investigated in many works, such as Boolean network [24–25], cooperative games [26], finite automata [27–29], multi-agent systems [30]. This paper investigates reachability problem of synchronizing transitions BPNSs in an algebraic form by using the STP of matrices.

The contributions of this paper are three-fold. First, we show how to represent the dynamics of a BPNS with synchronizing transitions in a hierarchical reachability graph (HRG) form, where the BPNS is combined with several bounded subnets via synchronizing transitions. The HRG that we present exploits the fact that both the

Received date: 18-08-2016
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DOI: 10.1016/S1005-8885(17)60190-0

dynamics and the reachable set (RS) of BPNSs can be expressed by its subnets. Second, accroding to the HRG, we present a new algebraic representation for the dynamics of the BPNSs with synchronizing transitions under the framework of the STP of matrices. The approach allow us to analyze the dynamical behavior of BPNSs in an algebraic form, especially for the big BPNSs in which the dynamics are not easily obtained. Third, we provide a necessary and sufficient condition for the reachability of BPNSs based on the algebraic form of dynamics, and present an algorithm for calculating the firing transition sequences between any two reachable markings. Our approach reduce the number of known markings in the process of calculation such that computational complexity is simplified when compared with the approach in Ref. [16].

The rest of this paper is organized as follows. Sect. 2 presents necessary notations and reviews the basic properties of the STP of matrices and PNs. In Sect. 3, the algebraic representation of BPNSs is characterized. In Sect. 4, for the BPNSs with synchronizing transitions, we define a new HRG, with which an algebraic expression for the dynamics is derived. We give the necessary and sufficient condition and the corresponding algorithm that allows us to verify the reachability between any two markings in Sect. 5. An illustration is provided to verify the proposed approach in Sect. 6. Finally, conclusions are shown in Sect. 7.

2 Preliminaries

2.1 Notations

In this section, we introduce some notations, which will be used in the sequel.

$\mathbb{R}^n$ is the set of n dimensional column vectors.

$\mathcal{M}_{m\times n}$ is the set of $m\times n$ real matrices.

$|\mathcal{M}|$ is the cardinality of the set $\mathcal{M}$.

$(\boldsymbol{A})_{(j)}$ is the jth column of the matrix $\boldsymbol{A}$ and we denote $C_{\mathrm{col}}(\boldsymbol{A})$ as the set of columns of matrix $\boldsymbol{A}$.

$\boldsymbol{\delta}_n^j, j=1,2,...,n$ is the jth column of the identity matrix $\boldsymbol{I}_n$. Particularly, let $\boldsymbol{\delta}_n^0 := [\underbrace{0,0,...,0}_{n}]^{\mathrm{T}}$. $\Delta_n := \{\boldsymbol{\delta}_n^1,...,\boldsymbol{\delta}_n^n\}$; $\tilde{\Delta}_n := \{\boldsymbol{\delta}_n^0,\boldsymbol{\delta}_n^1,...,\boldsymbol{\delta}_n^n\}$.

$\boldsymbol{A}\in\mathcal{M}_{m\times n}$ is called a logical matrix (or generalized logical matrix) if $C_{\mathrm{col}}(\boldsymbol{A})\subseteq\Delta_m$ (or $C_{\mathrm{col}}(\boldsymbol{A})\subseteq\tilde{\Delta}_m$). We denote the set of $m\times n$ logical matrices (or generalized logical matrices) by $\mathcal{L}_{m\times n}$ (or $\tilde{\mathcal{L}}_{m\times n}$).

If $\boldsymbol{A}\in\mathcal{L}_{m\times n}$ (or $\tilde{\mathcal{L}}_{m\times n}$), then it can be expressed as $\boldsymbol{A}=[\boldsymbol{\delta}_m^{i_1},...,\boldsymbol{\delta}_m^{i_n}]$ and its shorthand form, for simplicity, is $\boldsymbol{A}=\boldsymbol{\delta}_m[i_1,i_2,...,i_n]$, where $i_k\in\{0,1,...,m\}$, $k=1,2,...,n$.

2.2 STP of matrices

In this section, we give some necessary preliminaries on the STP of matrices, which will be used in the sequel. For more details, we refer the reader to Ref. [31].

Definition 1 The STP of matrices $\boldsymbol{A}\in\mathcal{M}_{m\times n}$ and $\boldsymbol{B}\in\mathcal{M}_{p\times q}$ is defined as [31]

$$\boldsymbol{A}\ltimes\boldsymbol{B}=(\boldsymbol{A}\otimes\boldsymbol{I}_{s/n})(\boldsymbol{B}\otimes\boldsymbol{I}_{s/p}) \tag{1}$$

where s is the least common multiple of n and p, $\otimes$ is the Kronecker product.

Remark 1 When $n=p$, the STP $\boldsymbol{A}\ltimes\boldsymbol{B}$ becomes the conventional matrix product $\boldsymbol{AB}$ where is denoted by $\boldsymbol{A}\ltimes\boldsymbol{B}=\boldsymbol{AB}$. In this paper, the notation '$\ltimes$' is mostly omitted hereafter since all the products of matrices are assumed to be the STP.

Lemma 1 For any two column vectors $\boldsymbol{X}\in\mathbb{R}^n$ and $\boldsymbol{Y}\in\mathbb{R}^m$, there is a unique swap matrix $\boldsymbol{W}_{[m,n]}\in\mathcal{L}_{mn\times mn}$ such that [31]

$$\boldsymbol{X}\ltimes\boldsymbol{Y}=\boldsymbol{W}_{[m,n]}\ltimes\boldsymbol{Y}\ltimes\boldsymbol{X} \tag{2}$$

where $\boldsymbol{W}_{[m,n]}=\boldsymbol{\delta}_{mn}[1,m+1,2m+1,...,(n-1)m+1,2,m+2,2m+2,\cdots,(n-1)m+2,...,m,2m,3m,...,nm]$.

Lemma 2 Given a logical vector $\boldsymbol{Y}\in\Delta_m$, there exists a unique power-reducing matrix $\boldsymbol{M}_r\in\mathcal{L}_{m^2\times m}$ such that [24]

$$\boldsymbol{Y}^2=\boldsymbol{M}_r\ltimes\boldsymbol{Y} \tag{3}$$

where $\boldsymbol{M}_r=\boldsymbol{\delta}_{m^2}[1,m+2,2m+3,...,m^2-m-1,m^2]$.

Lemma 3 For any two logical vectors $\boldsymbol{X}\in\Delta_m$ and $\boldsymbol{Y}\in\Delta_m$, there exists a unique dummy matrix $\boldsymbol{E}_{\mathrm{d}}\in\mathcal{L}_{m\times m^2}$ such that [32]

$$\left.\begin{aligned}\boldsymbol{E}_{\mathrm{d}}\boldsymbol{XY}&=\boldsymbol{Y}\\ \boldsymbol{E}_{\mathrm{d}}\boldsymbol{W}_{[m,m]}\boldsymbol{XY}&=\boldsymbol{X}\end{aligned}\right\} \tag{4}$$

where $\boldsymbol{E}_{\mathrm{d}}=[\underbrace{\boldsymbol{I}_m,...,\boldsymbol{I}_m}_{m}]$.

Lemma 4 Let $\boldsymbol{\delta}_{m^k}^d=\ltimes_{i=1}^{k}\boldsymbol{\delta}_m^{j_i}$ be the STP of k logical vectors $\boldsymbol{\delta}_m^{j_i}$ with $i=1,2,...,k$ [28]. Then we have $\boldsymbol{\delta}_m^{j_i}=\boldsymbol{S}_i^k\ltimes\boldsymbol{\delta}_{m^k}^d$, $i=1,2,...,k$, where