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Co-design of controller and routing redundancy over a wireless network ⋆

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Abstract: In this paper we investigate the exploitation of redundancy when routing actuation data to a discrete-time LTI system connected to the controller via a wireless network affected by packet drops. We assume that actuation packets can be delivered from the controller to the actuator via multiple paths, each associated with a delay and a packet loss probability. We show that the joint design of controller gain and routing redundancy exploitation can tremendously improve the control performance. To achieve this goal we set up and solve a LQR problem for a class of systems that extends discrete-time Markov Jump Linear Systems, in that both continuous and discrete control signals can be actuated.

Keywords: LQR Control Method, Wireless Communication Networks, Stochastic Control.

1. INTRODUCTION

Wireless control networks (WCN) are distributed control systems where the communication between sensors, actuators, and computational units is supported by a wireless communication network. The use of WCN in industrial automation results in flexible architectures and generally reduces installation, debugging, diagnostic and maintenance costs with respect to wired networks (see e.g. I.F. Akyildiz and I.H. Kasimoglu (2004), Han et al. (2010) and references therein). However modeling, analysis and design of (wireless) networked control systems are challenging open research problems since they require to take into account the joint dynamics of physical systems, communication protocols and network infrastructures. Recently, a huge effort has been made in scientific research on NCSs, see e.g. Aström and Wittenmark (1997), W. Zhang et al. (2001), G.C. Walsh et al. (2002), K.-E. Årzén et al. (2006), Tabbara et al. (2007), J.P. Hespanha et al. (2007), Gupta et al. (2009), M.C.F. Donkers et al. (2011), R. Alur et al. (2011), Pajic et al. (2011), D'Innocenzo et al. (2013) and references therein for a general overview.

To make a WCN robust to packets losses redundancy in data routing can be used. One approach to exploit this redundancy is relaying data via multiple paths and then appropriately combining them, which is reminiscent of network coding. In Smarra et al. (2015) we considered a state-feedback control loop as in Figure 1 where multiple copies $u_1(k) = u_2(k) = \ldots = u_r(k) = Kx_P(k)$ of the same actuation data are sent from the controller to the plant via r different routing paths $\{\rho_i\}_{i=1}^r$ each characterised by a delay d_i and a packet losses probability p_i. We assumed that the time-invariant controller gain K is designed via classical methods to assign the eigenvalues of the closed-loop system in the nominal case, i.e. when the effect of packet losses is not considered. We also assumed that the actuator computes a linear combination $\sum_{i=1}^r \gamma_i u_i(k-d_i)$ of the data incoming from different routing paths, and we provided a suboptimal algorithm to compute the optimal weights γ_i that maximize a metric induced by the notion of Mean Square Stability. In this paper we continue the research line started in Smarra et al. (2015) and provide novel results that strongly improve the controller performance. The first difference is motivated by the following consideration: when routing redundancy is exploited in communication systems the objective is to relay some information, and thus we send to the network the same packet $u_1(k) = \ldots = u_r(k)$ and try to extract from the corrupted received packets the original information; in our case the objective is to increase the control performance, as a consequence the actuation packets $u_1(k), \ldots, u_r(k)$ must not be necessarily equal. In this paper we perform *controller and routing redundancy co-design* by designing the time-varying matrix $\mathbb{R}^{rm\times n} \ni K(k) \doteq [K_1(k) \in \mathbb{R}^{m\times n}; \cdots ; K_r(k) \in \mathbb{R}^{m\times n}]$. Note that the problem formulation in Smarra et al. (2015) is a special case of the above definition where $K(k) = [\gamma_1 K^*; \cdots ; \gamma_r K^*]$ and K^* is designed for the nominal case. As a further improvement, while in Smarra et al. (2015) we optimise a metric based on the notion of Mean Square Stability (i.e. only taking into account the steady state behavior) we consider here a more complex control specification that also takes into account the transient behavior by setting up a finite-horizon LQR problem. In Mesquita et al. (2012) the authors also consider redundant data transmission over a set of paths characterised by i.i.d. Bernoulli probabilities of packet losses, but for a more restricted scenario with respect to ours because they assume that all paths are associated with the same delay, the packet loss events are measurable, the controller is designed for the nominal case (i.e. without considering the effect of packet losses) and redundant data combination

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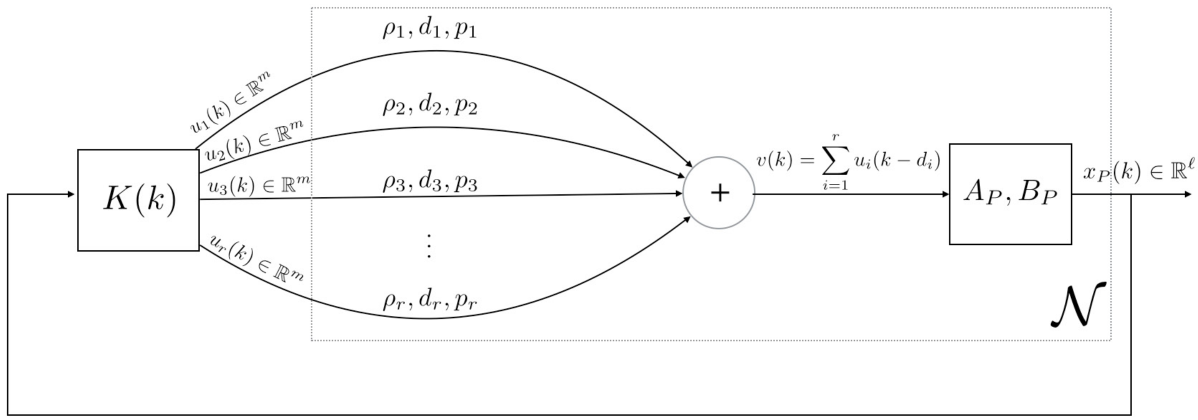

Fig. 1. State feedback control scheme.

is not modeled/designed. Their focus is on deciding how many redundant copies of a packet should be transmitted at each sampling time and what benefits can be drawn from this: besides the fact that our model is more general, we also address the more general problem of co-designing controller and routing redundancy.

In Section 2 we define a network modeling framework that allows co-design of controller and routing redundancy while taking into account the effect of packet losses and in Section 3 we provide our LQR problem formulation.

In Section 4 we assume that the set of paths used at each time instant to send actuation data to the actuator has be designed a priori: we will call this approach static routing redundancy. We setup the problem of co-desiging the controller gain and the routing redundancy parameters as a LQR problem for a Discrete-Time Markov-Jump Linear System (dtMJLS) where the discrete mode (which correpsond to the occurrence of packet losses) is unmeasurable and evolves according to a sequence of i.i.d. random variables. The latter assumption, widely adopted for several communication systems, makes particularly sense in our framework since exploitation of redundant data is well known to be very effective especially in the case when the reduntant paths used for data relay are uncorrelated from the point of view of the communication channels' characteristics. The proof for solving the LQR problem for such a system, being the discrete state unmeasurable, is an extension of the solution for dtMJLS in Costa et al. (2005) and can be derived without much difficulty thanks to the i.i.d. assumption. Note that a similar problem has been addressed in Matei et al. (2008), with the assumption that the discrete state is measurable with a one step delay, and solved without proof: we believe that the proof in this case is very close to ours, but since we were unable to find it in the scientific literature we provide one for the sake of completeness: in particular, because of space limitations, the reader is referred to G.D. Di Girolamo et al. (2015) for the proof of Theorem 1. We apply our optimal solution to a simple example where actuation data can be sent to the actuator via two paths: the first characterised by short delay (i.e. fast reaction to perturbations) and high probability of packet losses (i.e. low reliablity); the second characterised by long delay (i.e. slow reaction to perturbations) and 0 probability of packet losses (i.e. perfect reliablity). Note that such situation can often occur in realistic cases: one example is a multi-hop wireless network where we can reach the destination via a single long hop (short delay, high packet loss probability) or via a path of very short multiple hops (high delay, low packet loss probability); another example is a service provider network where we can reach the destination via the shortest yet congested path of routers (short delay, high packet loss probability) or via a longer uncongested path of routers (high delay, low packet loss probability). In the above situations, using only the first path is clearly not a good idea since the closed loop system may easily become unstable. Using only the second path is the optimal solution to maximise bandwidht, i.e. optimal from the point of view of communication theory: however, due to the high delay, the control system is not reactive to perturbations. The main idea that motivates this paper is based on the intuition that we could use both paths simultaneously, exploiting the fast reaction of the first path and the high reliability of the second path in an optimal way taking into account the plant dynamics. Our Monte Carlo (MC) simultations show that routing actuation data on both paths simultaneously and applying our optimal solution, we can tremendously improve the performance of both single-path solutions from the point of view of control performance.

In Section 5 we consider a much more complicated problem: we assume that the set of paths used at each time instant to send data to the actuator can be controlled, i.e. the choice of redundant routing paths is also a control variable: we will call this approach dynamic routing redundancy. We setup the problem of co-desiging the controller gain, routing redundancy parameters and paths as a LQR problem for a class of systems that includes dtMJLS as a special case and provide, as the main theoretical contribution of this paper, a recursive solution that is