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# Coordinated intersection traffic management

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**Abstract:** This paper considers the problem of coordinating the passage of vehicles through a traffic intersection with the aim of minimizing total travel time and energy consumption. The intersection manager communicates with vehicles heading towards the intersection, groups them into clusters (termed bubbles) as they appear, and determines an optimal order of passage and average velocity profiles. Vehicles in a bubble receive the corresponding profile and implement local control to avoid collision with other bubbles in the same road and within the bubble itself, and reach the intersection at the prescribed time and with the bubble occupying the intersection for no more than a prescribed duration.



*Keywords:* Intelligent transportation systems, hierarchical and distributed control, optimized operation and scheduling, state-based intersection management, networked vehicles

## 1. INTRODUCTION

Emerging technologies in intelligent transportation systems such as vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication have the potential to hugely impact safety, traveling ease, travel time, and energy consumption, eliminating road accidents and traffic collisions. A particularly useful application of these technologies is in the coordination of traffic at and near intersections. In contrast to traditional intersection management, networked vehicle technologies allow us to coordinate the traffic not just *within the intersection*, but also by controlling the vehicles' behavior much before they arrive at the intersection. Such a paradigm offers the possibility of significantly reduced stop times and increased fuel efficiency and is the subject of this paper.

### *Literature review*

Much of the literature in the area of coordination-based intersection management focuses on collision avoidance of vehicles *within the intersection*. Supervisory intersection management (intervention only when required to maintain safety by avoiding collisions) is explored using discrete event abstractions in (Dallal et al., 2013) and reachable set computations in (Colombo and Del Vecchio, 2015; Hafner et al., 2013). The works (Dresner and Stone, 2008; Fajardo et al., 2011) and references therein describe a multiagent simulation approach in which, upon a reservation request from a vehicle, an intersection manager accepts or rejects the reservation based on a simulation. Each vehicle attempts to conform to its assigned reservation and if this is prediced not to be possible at any time, the reservation is canceled. (Kowshik et al., 2011) also uses a reservation-based system to schedule intersection crossing times. In addition, the paper also provides provably safe maneuvers for vehicle following in a lane as well as for crossing the intersection. Hult et al. (2015); Campos et al. (2014) use model predictive control based method to coordinate the intersection crossing by vehicles and obtain suboptimal solutions to a linear quadratic optimal control problem. In (Qian et al., 2014) a heuristic policy assigns priorities to the vehicles, while each vehicle applies a priority-preserving control and legacy vehicles platoon behind a computer-controlled car.

We note that the ability to efficiently coordinate diminishes as the vehicles get closer to the intersection. This is why here we take an expanded view of intersection management that looks at the coordinated control of the vehicles much before they arrive at the intersection. The above methods are not suited for this setup or would prove to be too computationally costly. An example of the expanded view of intersection management is (Miculescu and Karaman, 2014), in which a polling-systems approach is adopted to assign schedules, and then optimal trajectories for all vehicles are computed sequentially in order. Such optimal trajectory computations are costly and depend on other vehicles' detailed plans, and hence the system is not robust. Closer to this paper, the works (Jin et al., 2012, 2013) describe a hierarchical setup, with a central coordinator verifying and assigning reservations, and with vehicles planning their trajectories locally to platoon and to meet the assigned schedule. The proposed solution is based on multiagent simulations and a reservation-based scheduling (with the evolution of the vehicles possibly forcing revisions to the schedule), both important differences with respect to our approach. (Li et al., 2014) is a recent survey of traffic control with vehicular networks and provides other related references.

### *Statement of contributions*

We propose a provably safe hierarchical intersection management system aimed at optimizing a combination of cumulative travel time and fuel usage. The proposed system is composed of three main aspects: (i) clustering to identify vehicles that must platoon before arriving at the intersection. We refer to such clusters of vehicles as *bubbles*; (ii) a branch-and-bound based scheduling algorithm that identifies the optimal schedule for a simplified cost function; (iii) a distributed control algorithm for the vehicles that ensures overall safety and guarantees that the actual intersection crossing schedule does not violate the prescribed schedule. Advantages of our proposed system include provably safe algorithms that do not require extensive simulations; dynamic clustering to account for

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the arrival of new vehicles in the problem domain and reduce the computational load on the branch-and-bound algorithm, a feature that also makes the algorithm applicable to a varied range of traffic conditions; and a distributed algorithm for local vehicular control which guarantees the desired aggregated behavior of each bubble. Proofs are omitted for reasons of space and will appear elsewhere.

## 2. PRELIMINARIES

We present here some basic notation and concepts on graph theory used throughout the paper.

### *Notation*

We let $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, $\mathbb{Z}$, $\mathbb{N}$, and $\mathbb{N}_0$ denote the set of real, nonnegative real, integer, positive integer, and nonnegative integer numbers, respectively. For a non-empty ordered list $\mathcal{S} = \{i_1, \ldots, i_s\}$, we let $|\mathcal{S}|$ denote the cardinality of $\mathcal{S}$. Further, $\mathcal{S}(i)$ denotes the $i^{\text{th}}$ element of $\mathcal{S}$. Thus, $\mathcal{S}(|\mathcal{S}|)$ denotes the last element of $\mathcal{S}$. For convenience, we also use the notation $j \in \mathcal{S}$ ($j \notin \mathcal{S}$) to denote that $j$ is (is not) an element of the ordered list $\mathcal{S}$. For two ordered lists $\mathcal{S}_1$ and $\mathcal{S}_2$, we let $\mathcal{S}_1 \setminus \mathcal{S}_2$ denote the ordered list of elements that belong to $\mathcal{S}_1$ but not to $\mathcal{S}_2$, while preserving the same order as in $\mathcal{S}_1$. We let the notation $[u]_{u_m}^{u_M}$ denote the number $u$ lower and upper saturated by $u_m$ and $u_M$ ($u_m \leq u_M$), respectively, i.e.,

$$[u]_{u_m}^{u_M} \triangleq \min\{u_M, \max\{u_m, u\}\}$$

### *Graph theory*

We review basic notions following the exposition in (Bullo et al., 2009). A digraph of order $n$ is a pair $G = (V, E)$, where $V$ is a set with $n$ elements called nodes and $E$ is a set of ordered pair of nodes called edges. A directed path is an ordered sequence of nodes such that any ordered pair of nodes appearing consecutively is an edge. A cycle is a directed path that starts and ends at the same node and that contains no repeated node except for the initial and the final one. A digraph is acyclic if it has no cycles. A directed (or rooted) tree is an acyclic digraph with a node, called root, such that any other node of the digraph can be reached by one and only one directed path starting at the root. If $(i, j)$ is an edge of a tree, $i$ is the parent of $j$, and $j$ is the child of $i$. Given a tree, a subtree rooted at $i$ is the tree that has $i$ as its root and is composed by all of its successors in the original tree.

## 3. PROBLEM STATEMENT

Consider an intersection and the incoming traffic along four branches as shown in Figure 1. For simplicity, we assume that (i) there is a single lane in each direction, (ii) all vehicles are identical with length $L$, (iii) vehicles do not turn at the intersection, (iv) there are no sources or sinks for vehicles along the branches - all new traffic appears at the beginning of the branches and must cross the intersection. It is possible to avoid assumption (iii) and allow turning. However, the differing travel speeds when turning and going straight affects the computation of the intersection occupancy time. In order to keep the problem setup and notation simpler, we make assumption (iii).

The dynamics of a vehicle with label $j$ is given by

$$\dot{x}_j^v(t) = v_j^v(t), \quad \text{(1a)}$$
$$\dot{v}_j^v(t) = u_j^v(t), \quad \text{(1b)}$$

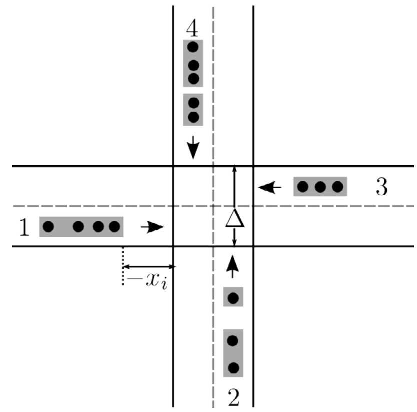


Fig. 1. Traffic near an intersection. Black dots represent individual vehicles, which are clustered and contained within *bubbles*, represented by grey boxes. $\Delta$ is the length of the intersection and the numbers $\{1, 2, 3, 4\}$ are labels for the incoming branches.

where $x_j^v$, $v_j^v \in \mathbb{R}$ are the position (negative of the distance from the front of the vehicle to the beginning of the intersection) and velocity of the vehicle, respectively and $u_j^v(t) \in [u_m, u_M]$, with $u_m \leq 0$ and $u_M \geq 0$, is the input acceleration. We use the superscript $v$ for the state and control variables of individual vehicles. We assume that each branch has a maximum speed limit that the vehicles must respect. Purely for the sake of simpler notation, we assume that the speed limit on all branches is the same and equals $v^M$. Thus, for each vehicle $j$, $v_j^v(t)$ must be constrained to belong to the interval $[0, v^M]$ for all time $t$ that the vehicle is in the system.

Each vehicle is equipped with vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communication capabilities. With the V2I communication, the vehicles inform a central *intersection manager* (IM) about their positions and velocities and receive from IM commands such as time to arrive at the intersection. We assume IM has the necessary communication and computing capabilities. We seek a design solution that minimizes a cost function $\mathcal{C}$, that models a combination of cumulative travel time and cumulative fuel cost, by scheduling the intersection crossing of the vehicles and controlling their approach to the intersection, all while avoiding collision. Solving this problem at the level of individual vehicles is computationally expensive and not scalable. Thus,we aim to synthesize a solution that makes this problem tractable to solve in real time and is applicable to a wide range of traffic scenarios.

## 4. OVERVIEW OF HIERARCHICAL SOLUTION

This section gives an outline of our hierarchical solution to the problem stated in Section 3. Our algorithmic solution combines optimized planning and scheduling of groups of vehicles with local distributed control to avoid collision and execute the plans, and has three distinct aspects,

(1) grouping the vehicles into clusters,
(2) scheduling the passage of the clusters through the intersection,
(3) local vehicular control to achieve and maintain cluster cohesion, to avoid collisions, and to ensure the clusters meet the prescribed schedule.

Each of these aspects is coupled with the other two. Moreover, an overarching theme is the dynamic nature of the problem due to the arrival and departure of vehicles.