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Design of Explicit Model Predictive Controllers Based on Orthogonal Partition of the Parameter Space: Methods and A Software Tool

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Abstract: This paper reviews the approaches to explicit approximate Nonlinear Model Predictive Control (NMPC), based on orthogonal partition of the parameter space. They are classified with respect to the type of control problem, the type of model describing the nonlinear system, the presence of uncertainty, the type of control input, and the type of the approximating control function. A software tool for design of explicit NMPC by applying the described approaches is presented. It is used to design an explicit NMPC controller for a continuous stirred tank reactor.

Keywords: Predictive control, Constraints, Nonlinear systems, Explicit solution, Software tools.

1. INTRODUCTION

For almost two decades, a great variety of methods for explicit solution of Model Predictive Control (MPC) problems have been developed (see for example the review in Alessio and Bemporad (2009)). The benefits of an explicit solution, in addition to the efficient on-line computations, include also verifiability of the implementation. This paper focuses on the review of those approaches to explicit MPC, which are based on orthogonal parameter space partitioning (Fig. 1) and their computer program implementation as a software package. First, a multi-parametric Quadratic Programming approach to explicit approximation of MPC problems for constrained *linear* systems has been introduced in Johansen and Grancharova (2003), which is based on orthogonal partitioning. A review of the methods for explicit *linear* MPC based on such partition can be found in Grancharova and Johansen (2005).

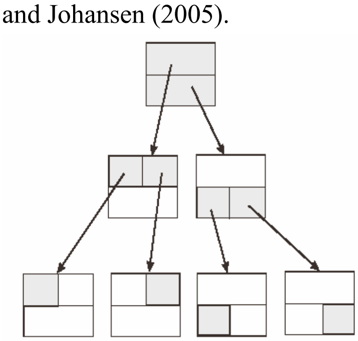

Fig. 1. $k-d$ tree partition of a rectangular region.

Later, in Johansen (2004), Grancharova and Johansen (2012), the parametric programming approach in Johansen and Grancharova (2003) has been extended to the explicit approximation of Nonlinear Model Predictive Control (NMPC) problems by applying the multi-parametric Nonlinear Programming (mp-NLP) ideas (Fiacco (1983)). In Summers et al. (2010), an algorithm for explicit NMPC is introduced based on multiresolution function approximation. The developed mp-NLP approaches have been applied to design explicit NMPC controllers for several practical systems (see Feng et al. (2010), Grancharova and Johansen (2011a), Grancharova and Johansen (2012)). It should be noted that the off-line computational complexity of the explicit NMPC increases very rapidly with the number of states and this would restrict its application only for systems with a few states. This has led to the development of several methods for complexity reduction of the explicit solution of NMPC problems (Grancharova and Johansen (2009)).

2. CLASSIFICATION OF THE APPROACHES TO DESIGN OF EXPLICIT NMPC CONTROLLERS

NMPC involves the solution at each sampling instant of a finite horizon optimal control problem subject to nonlinear system dynamics and state and input constraints (Mayne et al. (2000)). The approaches to explicit NMPC, based on orthogonal state space partition, can be classified as follows (Grancharova and Johansen (2011b)):

A. The type of the control problem

- **Regulation problem**

Consider the discrete-time nonlinear system:

$$x(t+1) = f(x(t), u(t)) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$ are the state, input and output variable, and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a nonlinear function. It is supposed that a full measurement of the state $x(t)$ is available at the current time t. Consider the optimal *regulation* problem, where the goal is to steer the system state to the origin. For the current $x(t)$, the *regulation* NMPC solves the optimization problem:

Problem P1:

$$V^*(x(t)) = \min_U J(U, x(t)) \quad (3)$$

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subject to $x_{t|t} = x(t)$ and:

$$y_{\min} \le y_{t+k|t} \le y_{\max}\,,\ k = 1, \dots, N \tag{4}$$

$$u_{\min} \le u_{t+k} \le u_{\max}\,,\ k = 0,1, \dots, N-1 \tag{5}$$

$$\left\| x_{t+N|t} \right\| \le \delta \tag{6}$$

$$x_{t+k+1|t} = f(x_{t+k|t}, u_{t+k})\,,\ k \ge 0 \tag{7}$$

$$y_{t+k|t} = C x_{t+k|t}\,,\ k \ge 0 \tag{8}$$

with $U = [u_t, u_{t+1}, \dots, u_{t+N-1}]$ and the cost function given by:

$$J(U, x(t)) = \sum_{k=0}^{N-1} \left[\left\| x_{t+k|t} \right\|_{Q_x}^2 + \left\| u_{t+k} \right\|_R^2 \right] + \left\| x_{t+N|t} \right\|_{P_x}^2 \tag{9}$$

In Johansen (2004), Grancharova and Johansen (2009), Grancharova and Johansen (2012), the explicit approximation of *regulation* NMPC problems is considered.

- **Reference tracking problem**

With the optimal *reference tracking* problem, the goal is to have the output vector $y(t)$ track the reference signal $r(t) \in \mathbb{R}^p$. For the current $x(t)$, the *reference tracking* NMPC solves the optimization problem:

Problem P2:

$$V^*(x(t), r(t), u(t-1)) = \min_U J(U, x(t), r(t), u(t-1)) \tag{10}$$

subject to $x_{t|t} = x(t)$ and:

$$y_{\min} \le y_{t+k|t} \le y_{\max}\,,\ k = 1, \dots, N \tag{11}$$

$$u_{\min} \le u_{t+k} \le u_{\max}\,,\ k = 0,1, \dots, N-1 \tag{12}$$

$$\Delta u_{\min} \le \Delta u_{t+k} \le \Delta u_{\max}\,,\ k = 0,1, \dots, N-1 \tag{13}$$

$$\left\| y_{t+N|t} - r(t) \right\| \le \delta \tag{14}$$

$$\Delta u_{t+k} = u_{t+k} - u_{t+k-1}\,,\ k = 0,1, \dots, N-1 \tag{15}$$

$$x_{t+k+1|t} = f(x_{t+k|t}, u_{t+k})\,,\ k \ge 0 \tag{16}$$

$$y_{t+k|t} = C x_{t+k|t}\,,\ k \ge 0 \tag{17}$$

with the cost function given by:

$$J(U, x(t), r(t), u(t-1)) = \sum_{k=0}^{N-1} \left[\left\| y_{t+k|t} - r(t) \right\|_{Q_y}^2 + \left\| \Delta u_{t+k} \right\|_R^2 \right] + \left\| y_{t+N|t} - r(t) \right\|_{P_y}^2 \tag{18}$$

In Grancharova and Johansen (2009), Grancharova and Johansen (2011a), Grancharova and Johansen (2012), the explicit approximation of *reference tracking* NMPC problems is considered.

In problems P1, P2, N is a finite horizon and $P_x, Q_x, P_y, Q_y, R \succ 0$. In problem P1 it is assumed that $f(0,0) = 0$ and in problem P2 it is supposed that $\Delta u_{\min} < 0 < \Delta u_{\max}$. From a stability point of view it is desirable to choose δ in (6) or in (14) as small as possible (Mayne et al. (2000)). However, the feasibility of problems P1 and P2 will rely on either δ or N being sufficiently large. A part of the NMPC design is to address this tradeoff.

B. The type of the model

- **Explicit NMPC based on first principles models**

Most often, the NMPC problem formulation is based on *first-principles models* of the general form (1)–(2). For example, these models can be derived from energy and mass balances for technological processes (e.g. chemical production processes), the laws of motion and air dynamics for pneumatic systems, the equations of motion for robots etc. The design of explicit NMPC controllers for systems described by first principle models is considered in Johansen (2004), Grancharova and Johansen (2009), Feng et al. (2010), Grancharova and Johansen (2011a), Grancharova and Johansen (2012). In Grancharova and Johansen (2011a), two types of explicit reference tracking NMPC controllers for an electro-pneumatic clutch actuator using on/off valves are designed and compared. In Feng et al. (2010), an mp-NLP approach is applied to construct an explicit solution of the optimal decompression profiles of divers.

- **Explicit NMPC based on black-box models**

Some times it is more convenient to describe the system dynamics by using *black-box models* (neural network models, fuzzy models, local model networks, Gaussian process models). In Grancharova et al. (2011), Grancharova and Johansen (2012), a general approach for design of explicit output-feedback NMPC controller based on black-box models is suggested. It is supposed that the dynamics of the nonlinear system is described by a NARX model:

$$y(t+1 \mid \theta) = f_{BB}(z(t), u(t), \theta) \tag{19}$$

where f_{BB} is the function realized by the black-box model, θ contains the model parameters, and $z(t)$ is the regressor vector, defined by:

$$z(t) = \begin{cases} [y(t), y(t-1), \dots, y(t-L), \\ \quad u(t-1), \dots, u(t-L)]\,, \text{ if } L > 0 \\ y(t)\,, \text{ if } L = 0 \end{cases} \tag{20}$$

Here, L is a given lag, and (19)–(20) are defined for $t \ge L$. Further, in Grancharova et al. (2011), a dual-mode control strategy is employed in order to achieve an offset-free closed-loop response in the presence of bounded disturbances and/or model errors. The dual-mode approach is applied to design an explicit NMPC for regulation of a pH maintaining system, described by *a neural network NARX model*. In Grancharova et al. (2008), Grancharova and Johansen (2012), an approximate mp-NLP approach to explicit solution of stochastic NMPC problems for constrained nonlinear systems based on a *Gaussian process model* is developed. It is applied to design an explicit stochastic reference tracking NMPC controller for a combustion plant.

C. The presence of uncertainty

It is supposed that the nonlinear system is described by:

$$x(t+1) = f(x(t), u(t), w(t)) \tag{21}$$

$$y(t) = h(x(t), u(t), w(t)) \tag{22}$$

where $w(t) \in \mathbb{R}^s$ are the uncertainty variables. In the *robust* NMPC problem formulation, the model uncertainty is taken into account. There are two possible cases:

- **Polyhedral description of the uncertainty**

In this case, the only information available for the uncertainty is that it belongs to a bounded polyhedral set, i.e. $w(t) \in W^A \subset \mathbb{R}^s$. Let $W = [w_t, \dots, w_{t+N-1}] \in W^B$, with $W^B = W^A \times \dots \times W^A \subset \mathbb{R}^{Ns}$, denote an uncertainty realization. For the current $x(t)$, the *robust* NMPC minimizes the worst-case cost function value through the following optimization: