TIME DELAY ROBUST CONTROL - PROGRAM IMPLEMENTATION

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Abstract: The contribution presents a control design method for robust tuning of continuous-time controllers for SISO with time delays. Controllers are obtained via general solutions of Diophantine equations in the ring of proper and stable rational functions. The methodology covers both stable and unstable systems. The control objectives are obtained through divisibility conditions in this ring. Robustness of proposed algorithms can be studied through the infinity norm and sensitivity function. Several approximations of time delay terms and two control structures of controlled loop were considered. A scalar parameter was proposed as a tuning knob for influencing of stability, minimization of the sensitivity function and uncertainty evaluation. A Matlab-Simulink package was developed for automatic design and simulation of proposed algorithms.

Key words: Rings, time delays, SISO systems, robust control, PID controllers.

1. INTRODUCTION

Continuous-time controllers of the PID type have been widely used in many industrial applications for decades. There are several features to their success, e.g. structure simplicity, reliability, robustness in performance, see e.g. (Aström and Hägglund, 1995; Rad and Lo, 1992). However, the choice of the individual weighting of the three actions, i.e. proportional, integral and derivative has been a problem. Moreover, the number of tuning parameters in more sophisticated PID modifications proposed in (Aström, *et al*., 1992) is even higher. Robust controllers and plant uncertainty have become a requisite and popular notions in control theory during for many years. Robustness also influenced design and tuning of PID controllers (Morari and Zafiriou, 1989; Prokop and Corriou, 1997). The necessity of robust control was naturally developed by the situation when the nominal plant (used in control design) differs from the real (perturbed) one. A suitable tool for parameter uncertainty is the infinity norm H_∞. Hence, a polynomial description of transfer functions had to be replaced by another one. A convenient description adopted from (Vidyasagar, 1985; Kučera 1993; Doyle, *et al*., 1992) is a factorization approach where transfer functions are expressed as a ratio of two Hurwitz stable and proper rational functions. Then, the conditions of robust stability can be easily formulated in algebraic parlance and all controllers are obtained and parameterized via linear Diophantine equations in an appropriate ring. Moreover, further tasks as disturbance rejection can be easily solved through the divisibility conditions of all stabilizing controllers.

A useful and interesting application of the mentioned robust philosophy can be seen in time delay systems. The dynamics of many technological plants can be adequately approximated by first order transfer functions plus dead time. This transfer function can be stable or unstable one. The situation where linear systems have unstable poles may occur e.g. in

a continuous-time stirred exothermic tank reactor, in polymerisation processes or in a class of biochemical processes where the processes must operate at an unstable steady state. Moreover, a time delay is an inherent part of many technological plants.
For SISO systems of the first and second order this approach yields a class of PID like controllers. The methodology is proposed and analysed in (Prokop and Mészáros, 1996; Prokop and Corriou, 1997; Prokop and Prokopová, 1998; Prokop, *et al.*, 2001). The algebraic approach gives for two degree of freedom structure a nontraditional PID controller proposed in e.g. (Aström, *et al.*, 1992; Morari and Zafiriou, 1989) in a different way. The fractional approach for SISO controllers brings a scalar parameter $m > 0$ which strongly influences the dynamic of the feedback system as well as the robustness and sensitivity of proposed controllers.

2. THEORETICAL BACKGROUND

Any transfer function $G(s)$ of a (continuous-time) linear system has been traditionally expressed as a ratio of two polynomials in s. For the purposes of this contribution it is necessary to express the transfer functions as a ratio of two elements of $R_{PS}(s)$. It can be easily performed by dividing, both the polynomial denominator and numerator by the same stable polynomial of the order of the original denominator. Moreover, a scalar parameter $m>0$ seems to be a suitable „tuning parameter" influencing control behaviour as well as robustness of the closed loop system. Then all transfer functions could be described by

$$G(s) = \frac{b(s)}{a(s)} = \frac{\frac{b(s)}{(s+m)^n}}{\frac{a(s)}{(s+m)^n}} = \frac{B(s)}{A(s)}, \qquad (1)$$
$$n = \max(\deg(a), \deg(b)), \quad m > 0$$

Time delay systems with "pure dead time" will be considered as system (1) in the form

$$G(s) = \frac{\frac{b(s)}{(s+m)^n} e^{-\tau s}}{\frac{a(s)}{(s+m)^n}} = \frac{B(s)}{A(s)}, \qquad (2)$$
$$\tau > 0, \; m > 0$$

Note that the divisibility in the ring $R_{PS}(s)$ is defined through all unstable zeros of elements in the ring. More precisely, element A divides element B in $R_{PS}(s)$ iff all zeros including infinite ones of A are also zeros of B. The H_∞ norm in the ring $R_{PS}(s)$ is defined by

$$\|G\| = \sup_{\mathrm{Re}\, s \geq 0} |G(s)| = \sup_{\omega \in E} |G(j\omega)|$$
$$\|G_1; \; G_2\| = \left\| \begin{matrix} G_1 \\ G_2 \end{matrix} \right\| = \sup_{\mathrm{Re}\, s \geq 0} \{|G_1(s)|^2 + |G_2(s)|^2\}^{\frac{1}{2}} \qquad (3)$$

This (called infinity) norm is the radius of the smallest circle containing the Nyquist plot of the transfer function and it is a convenient tool for the evaluation of uncertainty. The distance of two elements in the ring can be easily expressed by this norm. Let $G(s) = B(s)/A(s)$ be a nominal plant and consider a family of perturbed systems $\tilde{G}(s) = \tilde{B}(s)/\tilde{A}(s)$ where

$$\|A - \tilde{A}\| \leq \varepsilon_1 \;, \; \|B - \tilde{B}\| \leq \varepsilon_2$$
$$or \quad \|A - \tilde{A}; \quad B - \tilde{B}\| \leq \varepsilon \qquad (4)$$

where ε_1, ε_2, ε are positive constants.

3. APPROXIMATION OF TIME DELAY TERMS

Systems with time delay are obtained by an appropriate approximation of the term $e^{-\Theta s}$ see e.g. (Prokop, *et al.*, 1997). The approximation for time delay processes can be shown by a first order model

$$G_1(s) = \frac{Ke^{-\tau s}}{s+\alpha} \qquad (5)$$

The value of the parameter $\alpha > 0$ represents stable systems, $\alpha < 0$ represents unstable ones and $\alpha = 0$ is an integrator. For linear control design, it is necessary to approximate the time delay term in (5). It can be done in several methods. The simplest case is to neglect the delay term $e^{-\tau s}$. Then the time delay is considered as a perturbation of a nominal transfer function. So the first nominal approximation is

$$G_2(s) = \frac{K}{s+\alpha} \qquad (6)$$

Next two approximations are based on the Taylor series approximation of $e^{-\tau s}$ in the numerator or in the denominator. Approximations $e^{-\tau s} \approx (1-\tau s) \approx (1+\tau s)^{-1}$ then give

$$G_3(s) = \frac{K(1-\tau s)}{s+\alpha} = \frac{b_1 s + b_0}{s+\alpha} \qquad (7)$$

$$G_4(s) = \frac{K}{(s+\alpha)} \frac{1}{(1+\tau s)} = \frac{b_0}{s^2 + a_1 s + a_0} \qquad (8)$$

The last model can be obtained by the traditional Padé approximation

$$G_5(s) = \frac{K}{s+\alpha} \frac{1 - \frac{\tau}{2} s}{1 + \frac{\tau}{2} s} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \qquad (9)$$

All approximated transfer function can be written in R_{ps} in the form