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Joint Inventory and Pricing Decisions

John G. Wilson* Chris K. Anderson**

**Ivey Business School, Western University, London, ON N6G 0N1*
CANADA (Tel: 519-661-3867; e-mail: jwilson@ivey.uwo.ca).
***Cornell University, School of Hotel Administration, Ithaca, NY 14853*
USA (Tel: 607-255-8687; email: cka9@cornell.edu)

Abstract: The problem of assigning inventory to different pricing levels is considered. The problem is motivates by hoteliers assigning rooms to an opaque discounter. It can also be thought of as assigning seats and fares to two different classes where the cheaper class sells out first. The approach can be used for retail promotions such as "all items 20% of during the first hour of business". While the operations literature has looked extensively at joint pricing and inventory decisions in the single product setting, we extend the literature and provide closed form solutions to the multiproduct setting where demand across the products is dependent and the products share resources.

Keywords: Opaque channel, newsvendor, revenue management, discounting, pricing.

1. INTRODUCTION

The problem considered here is motivated by a certain group of hoteliers placing rooms on the internet site *Priceline*. On this site, users provide their credit card information and place a bid on a room at a certain star level for a particular region on a particular day. The customer does not know the name of the hotel that might offer the room or the precise location. Once a bid is accepted, the customer's credit card is charged. The customer is not technically allowed to bid again. This opaque mechanism for disposing of inventory has been the subject of academic research. Wilson and Zhang, 2008, consider how *Priceline* might optimally design such an auction so that each customer will make the maximum bid. Fay, 2004, considers the case of more than one bid. A review of approaches to modelling name-your-own-price auctions can be found in Anderson and Wilson, 2011

Priceline asks its suppliers to supply a number of rooms at different price levels. The room level may be the same but the price is different. For instance, the hotel might supply 15 four star rooms…10 at the price of $100 and 5 at the price of $150. For its non-preferred providers—the focus of this paper—*Priceline* will accept a bid if it is above $100 but will only remit $100 dollars to the hotel no matter how high the bid is until the 10 room quota is exhausted. Only then –if a bid is above $150-will the hotel receive $150. For instance, if a bid of $200 is received and some of the 10 rooms remain, then *Priceline* receives $100 and the hotel receives $100. If the 10 room quota is exhausted a bid of $200 would result in $50 to *Priceline* and $150 to the hotel. For the hotel, deciding on these quotas can be important. Descriptions of the process used by *Priceline* can be found in Anderson, 2009, Anderson and Xie, 2012 and Anderson et al., 2014. This last paper introduces the non -preferred provider problem and uses a Markov modelling approach to provide numerical results.

A review of pricing for the newsvendor problem can be found in Pertruzzi and Dada, 1999. Optimizing both inventory and price when demand is random has proven to be difficult. Raz and Porteous, 2006 assume that demand can be approximated by a deterministic model with random states. Wilson and Sorochuk, 2009, reduce the problem to a one dimensional numerical optimization. Moving to more than one level of inventory will clearly add considerable complexity.

The problem also has analogies to airline revenue management. In the approaches taken there to allocate seats to various fare classes, the assumption is usually made that high paying customers arrive first (see, e.g., Talluri and Van Ryzin 2005). Here, the assumption is that all lower fares sell first. In this paper, we assume that all customers arrive at random. Unlike traditional open outcry auctions, in the online auction setting bidders arrive throughout the auction duration and suppliers need to decide bid acceptance policies (price) and inventory allocations prior to observing all bids.

The operations and supply chain management literature have also focused on auction settings for product procurement. Emiliani, 2000 provides a review of early business-to-business (B2B) use of online auctions for purchasing and discusses open research issues. Priceline's model is the standard reverse auction and works well in settings with multiple suppliers and a single buyer. This is also the standard model used in many B2B auction enabled purchasing settings where large firms, e.g. General Motors use online auctions to secure largely commoditized inputs.

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2. MODELING APPROACH

The general case is to assume that demand X_p at a given pricepoint p has a density or mass function represented by $f_{X_p}(.)$ and that the $X_p$are independent random variables. We assume that the items are goods such as excess inventory or unsold seats where we can neglect the per item cost. This is common in the literature and in the hotel case is a very reasonable assumption: costs are fixed and every extra room sold to a discounter incurs negligible extra cost. For the airline case, an extra passenger can result in extra fuel costs…although this is not often included in the revenue management models. The goal is to find $N_1, N_2,\ldots N_m$ inventory levels at associated prices of $p_1 > p_2 > \cdots p_m$ respectively, to maximize expected revenue. No matter what the bid, the hotel will only receive a price of p_i if all inventory levels assigned to lower prices are exhausted.

In practice, hotels rarely use more than two or three levels. So the preliminary analysis of this paper is to assume only two levels. At this stage the two prices will be fixed at p_1 and p_2.

To simplify notation let *X* denote the demand at the higher price and *Y* the demand at the lower price. Suppose that the hotel assigns *c* rooms to the lower price and there are *x* people willing to buy at price p_1 and *y* willing to buy at price p_2, where $p_2 < p_1$ and that the first *c* sales are at price p_2. (The quantity *c* will ultimately be the decision variable.) For sales to be made at the price p_1, it must be the case that $x + y > c$. Since the customers are assumed to arrive at random, the number of sales at p_1 is random. Assuming that $+y > c$, one can think of the last $x + y - c$ sales at p_1 being a random choice of $x + y - c$ items from $x + y$ where the number of "successful" possibilities equals *x*, i.e. as a choice from a hypergeometric random variable where the expected number of successful draws equals $(x + y - c)(\frac{x}{x+y})$. Thus the expected revenue knowing the values *x* and *y* equals $p_2 \min(c, x + y) + p_1 \max(x + y - c, 0)(\frac{x}{x+y})$.

Of course, the numbers that arrive are random and the expected revenue from an assignment of *c* to the price p_2 equals $p_2\,\mathrm{E}[\min(c, X + Y)] + p_1 \mathrm{E}[\max(X + Y - c, 0)\left(\frac{X}{X+Y}\right)]$.

$\mathrm{E}[\min(c, X + Y)]$

$$= \iint_{x+y<c} (x + y) f_{X,Y}(x, y)dxdy + c \iint_{x+y\ge c} f_{X,Y}(x, y)dxdy$$

$$\mathrm{E}\left[\max(X + Y - c, 0)\left(\frac{X}{X + Y}\right)\right]$$
$$= \iint_{x+y\ge c} \frac{x}{x + y} f_{X,Y}(x, y)dxdy$$

A further assumption—common in much of the literature—is to assume uniform distributions for their tractability and usefulness in modeling a number of realistic situations. Using these distributions will result is closed form results and the derivation of useful properties and insight. Of course, in practice, one must be careful about making assumptions that are too divergent from reality (see Wilson et al, 211, for a discussion of this.) So assume that *X* is uniform over the range (0,*a*) and *Y* is uniform over the range (0,*b*). It makes sense to assume that $a < b$ as otherwise there would be people willing to buy at the higher price and not at the lower price.

There are three cases to be considered: $0 \le c < a$, $a \le c < b$, and $b \le c \le a + b$.

The expected value of *X*+*Y* when this sum is less than *c* is given by

$$\iint_{x+y<c} (x + y)/(ab)dxdy$$
$$= \begin{cases} \dfrac{c^3}{3ab} \; for\; 0 \le c < a \\ \dfrac{3ac^2 - a^3}{6ab} \; for\; a \le c < b \\ \dfrac{-2c^3 + (3b + 3a)c^2 + b^3 + a^3}{6ab} \; for\; b \le c \le a + b. \end{cases}$$

The probability that the total number is at least *c* is given by

$$\iint_{x+y\ge c} (\frac{1}{ab})dxdy$$
$$= \begin{cases} 1 - \dfrac{c^2}{2ab} \; for\; 0 \le c < a \\ \dfrac{a^2 + 2ab - 2ac}{2ab} for\; a \le c < b \\ \dfrac{c^2 - 2(b - a)c + (b + a)^2}{2ab} for\; b \le c \le a + b. \end{cases}$$

The above expression relates to obtaining the price p_2,. The expected number of sales at the price p_1 is as follows:

$$\iint_{x+y\ge c} \frac{(x + y - c)x}{(x + y)ab} dxdy$$
$$= \begin{cases} \dfrac{c^3 + (-d_3 + d_4)c + 6a^2b}{12ab} for\; 0 \le c < a \\ \dfrac{6a^2clogc - d_4c + 6a^2b + 4a^3}{3ab} \; for\; a \le c < b \\ \dfrac{(d_1clogc - c^3 - (d_3 + d_5)c + d_6}{12ab} \; for\; b \le c \le a + b, \end{cases}$$

where $d_1 = 6(a^2 - b^2), d_2 = 6a^2b + 4a^3, d_3 = d_1 \log(a + b), d_4 = -6b^2logb - 6ab + 6a^2loga, d_4 = d_3 + 6b^2logb - 6ab - 3a^2, d_5 = 3b^2 + 6ab + 3a^2, d_6 = 6b^2 - 2b^3 + 6a^2b + 4a^3$.