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Material Requirement Planning under Fuzzy Lead Times

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Abstract: This paper proposes a fuzzy multi-objective integer linear programming approach to model a material requirement planning (MRP) problem with fuzzy lead times. We incorporate to the crisp MRP model the possibility of occurrence of each one of the possible lead times. Then, an objective function that maximizes the possibility of occurrence of the lead times is considered. By combining this objective with the initials of the MRP model, decision makers can play with their risk attitude of admitting lead times that improve the other objectives but have a minor possibility of occurrence.

Keywords: Material requirement planning, uncertainty modelling, lead times, fuzzy modeling, multi-objective optimisations.

1. INTRODUCTION

There are many forms of uncertainty that could affect material requirement planning (MRP) systems. Ho (1989) identifies two uncertainty groups: (i) environmental uncertainty, which includes uncertainty in demand and supply; and (ii) system uncertainty, which is related to operation yield uncertainty, production lead time uncertainty, quality uncertainty, failure of production system and changes to product structure. This leads to the development of models for MRP with uncertainty (Mula et al. 2006).

MRP models under uncertainty in demand are the main addressed by the scientific literature through stochastic modelling (Escudero and Kamesam, 1993), fuzzy mathematical programming (Mula et al. 2006; Mula et al. 2007; Mula et al. 2008), safety stocks (Grubbström and Tang, 1999; Mula et al. 2014) or safety times (Wijngaard and Wortmann, 1985). Other approaches can be found in Mula et al. (2006).

With respect to MRP models under uncertain in lead times, it is necessary to highlight the seminal works by Yano (1987a, b, c) based on stochastic lead times and also the works by Dolgui and Louly (2002) and Louly and Dolgui (2004). Other approaches can be found in Dolgui and Prodhon (2007), Dolgui et al. (2013) and Aloulou et al. (2014).

This paper proposes a fuzzy multi-objective decision model for the material requirement planning (MRP) problem. Here, the main contribution is to provide an initial solution methodology to address MRP problems with fuzzy lead times. In order to validate the model, a numerical example is presented to illustrate the proposed solution methodology

2. FUZZY MULTI-OBJECTIVE MODEL FORMULATION FOR MATERIAL REQUIREMENT PLANNING

2.1 Assumptions

The following assumptions have been considered.

- A multi-product manufacturing environment. By the term product we refer to finished goods, components, raw materials and subassemblies structured in a bill of materials.
- A multi-level production systems where the subsets of components are assembled independently.
- A multi-period planning horizon comprised of a set of consecutive and integer time periods of the same length.
- The lead time of a product is the number of consecutive and integer periods that are required for their finalization.
- The inventory of each product (finished good, raw materials and components) is the available volume at the end of a given period.
- The backlog of the demand of a product at the end of a period is defined as the non negative difference between the cumulated demand and the volume of available product.
- The master production schedule (MPS), that specifies the quantity to produce of each finished good in every period of the planning horizon, and the MRP, that provides the net requirements of raw

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materials and components for each planning period, are solved jointly.

- Programmed receptions.
- Production capacity constraints.
- Overtime limits.
- It is assumed that the subcontracted products will be ready just when required without lead time changes.
- Fuzzy lead time for finished goods, components and raw materials.
- Fuzzy lead times are represented by using different values associated with different degree of possibility each one.

2.2 Fuzzy objective functions

Three fuzzy objective functions have been considered: (1) minimizes the total costs over the time periods that have been computed; (2) minimizes the backorder quantities over the whole planning horizon; and (3) minimizes the idle time of the productive resources.

$$Min \quad z_1 \cong \sum_{i=1}^{I}\sum_{t=1}^{T}\left(cp_{it}P_{it} + ci_{it}INVT_{it}\right) + \sum_{r=1}^{R}\sum_{t=1}^{T} ctov_{rt}Tov_{rt} \tag{1}$$

$$Min \quad z_2 \cong \sum_{i=1}^{I}\sum_{t=1}^{T} B_{it} \tag{2}$$

$$Min \quad z_3 \cong \sum_{r=1}^{R}\sum_{t=1}^{T} Tun_{rt} \tag{3}$$

2.3 Constraints

The following constraints have been included.

$$INVT_{i,t-1} + P_{i,t-LT_i} + SR_{it} - INVT_{i,t} - B_{i,t-1} - \sum_{j=1}^{I}\alpha_{ij}(P_{jt} + SR_{jt}) + B_{it} = d_{it} \qquad \forall i \forall t \ (4)$$

$$\sum_{i=1}^{I} P_{it}AR_{ir} + Tun_{rt} - Tov_{rt} = CAP_{rt} \qquad \forall r \forall t (5)$$

$$B_{iT} = 0 \qquad \forall i \ (6)$$

$$P_{it}, INVT_{it}, B_{it}, Tun_{it}, Tov_{it} \geq 0 \qquad \forall i \forall r \forall t \ (7)$$

$$P_{it}, INVT_{it}, B_{it} \in \mathrm{Z} \qquad \forall i \forall t \ (8)$$

Constraint (4) is the inventory balance equation for all the products. Constraint (5) establishes the available capacity for normal, overtime and subcontracted production. Constraint (6) finishes with the delays in the last period (T) of the planning horizon. Constraint (7) contemplates the non negativity for the decision variables and constraint (8) establishes the integrity conditions for some of the decision variables.

3. SOLUTION METHODOLOY

Here, an approach to transform the fuzzy goal programming (FGP) into an equivalent auxiliary crisp mathematical programming model for MRP problems is provided. This approach considers non increasing linear membership functions for each fuzzy objective function as follows (Bellman and Zadeh 1970):

$$\mu_k = \begin{cases} 1 & z_k < z_k^l \\ \dfrac{z_k^u - z_k}{z_k^u - z_k^l} & z_k^l < z_k < z_k^u \\ 0 & z_k > z_k^u \end{cases} \tag{9}$$

where μ_k is the membership function of z_k, while z_k^l and z_k^u are, respectively, the lower and upper bounds of the objective function z_k.

The FGP approach by Torabi and Hassini (2008), based on the convex combination of the lower bound for satisfaction degree of objectives and the weighted sum of these achievement degrees, is adopted as the basis of this solution methodology. This FGP programming method proposes that a multi-objective model could be transformed in a single objective model as follows:

Max

$$\lambda(x) = \gamma\lambda_0 + (1-\gamma)\sum_k \theta_k \mu_k(x)$$

subject to

$$\lambda_0 \leq \mu_k(x) \quad k = 1,\ldots,n$$

$$x \in F(x) \tag{10}$$

where μ_k represents the satisfaction degree of the k th objective function. $\lambda_0 = \min\{\mu_k(x)\}$ is the minimum satisfaction degree of the objectives. θ_k is the relative importance of the kth objective and γ is a coefficient of compensation.

Then, the equivalent auxiliary crisp mathematical programming model is formulated as follows: