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Effects of inventory control on bullwhip in logistic systems under demand and lead time uncertainties

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Abstract: In this paper, we are interested on the inventory control problem of multi-stages supply chain characterized by finite capacities of resources and delays due to processing times. The objective is to define a control law which permits to satisfy the end-customer demands and for which supply chain requirements will be completely met. Our model includes two factors that commonly have an impact on the supply chain performances and cause the bullwhip effect: the variability of the customer demand and the uncertainty on the lead time. The developed approach is based on a saturated predictor-feedback structure. We examine then the bullwhip effect phenomenon, which is an important observation in supply chain management. The results demonstrate that the bullwhip effect can be reduced, but not completely eliminated, by using a saturated command and a predictor-feedback structure.

Keywords: Inventory control, variability customer demand, lead time uncertainty, bullwhip effect, delayed system, saturated command, predictor-feedback structure.

1. INTRODUCTION

This paper deals with the problem of inventory control for constrained supply chain within uncertainties on the lead times and under unknown customer demand. This demand is rapidly variable and, since the tasks in the supply chain take significant times, this variation generates oscillations on inventory levels. In (Forrester, 1973), the author noted that small changes in demand are amplified along the supply chain, leading to larger variations in demand supported by the different levels, as they are further away from the customer. This is called the *Bullwhip Effect*. This phenomenon is due to four main causes: variation of the demand forecasting, order batching, shortage gaming, and price fluctuations. (Disney and Towill, 2003) demonstrated that the bullwhip effect leads the supply chain to unnecessary (undesired) costs that can represent more than 30%, in some cases, of the total costs thereof.

In this work, we take account two factors that commonly cause the bullwhip effect: the variability of the customer demand and the uncertainty on the lead times. The objective is the reduction of bullwhip effect, which is generated by the amplification of the variability of orders along the supply chain, from the customer to the factory. For that, we must first determine control law defining the execution orders in order to satisfy the customer demand taking into account all the constraints of the system. A common approach is to use the MRP techniques ((Hnaien et al., 2008),(Louly et al., 2008)). The problem of calculation of planned lead times under lead time uncertainties was studied with fixed due dates for the customer demand. Based on the demand and taking account the fixed lead time, replenishment orders are calculated for a series of discrete time intervals. The criterion considered is the sum of backlogging and holding costs using discrete optimization methods. However, these techniques supposed that the demand is known exactly. In this study, we assume that customer demand is unknown and lead times are considered with an uncertainty. Our objective is to design a control law and to reduce the bullwhip effect. Because of the presence of the delays, the supply chain is represented as *delayed systems* (Loiseau et al., 2009). The control law proposed is based on saturated and predictor-feedback structure, which is a classical method to control time-delay systems ((Riddalls and Bennett, 2002)). The paper is organized as follows. In section 2, the problem statement is described. Section 3 deals with the inventory control for a single logistic system. In section 4, the proposed approach is generalized for the multi-stages supply line. After determining the control law, we analyze in section 5 the impact of the inventory control on the bullwhip effect with simulation examples. We conclude the paper with discussions of using the proposed approach and give directions for future work.

2. PROBLEM STATEMENT

2.1 Model Description

We consider a multi-stages supply chain, composed of N stages, as illustrated on the figure 1. It consists of a series of stages called nodes. Each node represents either manufacturers, suppliers, distributors, or other actors in the supply chain that are involved in the process providing, in which different flows are exchanged between nodes in order to satisfy end customers requests. Each stage specified with a subscript i, $i = 1,...,N$, represents a basic logistic system composed of a supplying unit and a storage one. The supplying unit is characterized by a supplying order rate denoted $u_i(t)$ limited by a maximum supplying capacity denoted U_{max_i}, and by a delay θ_i. The storage

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unit is characterized by the inventory level denoted $y_i(t)$ and by a maximum storage capacity denoted Y_{max_i}. In this work, the end-customer demands denoted $d_c(t)$ are supposed to be unknown in advance but assumed to be upper bounded by an amount denoted D_{max}. We assume that $u_i(t)$, $y_i(t)$ and $d_c(t)$ are time-continuous functions.

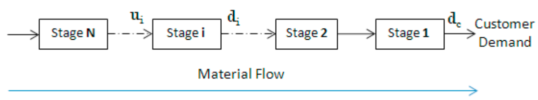

Fig. 1. Multi-stages supply chain model.

In such serially linked structure, each stage i has one supplier $u_i(t)$, and is supposed to support the incoming demand $d_i(t)$ of the following stage such that $d_i(t) = u_{i-1}(t)$, for $i = 2, ..., N$. The first stage indexed $i = 1$ corresponds to the final retailer which faces the end consumer demand $d_c(t)$. The inventory dynamics of each stage is given by the following equation, $i = 1, ..., N$.

$$\dot{y}_i(t) = \begin{cases} u_i(t-\theta_i) - d_i(t) & \text{for } t \geq \theta_i \\ \phi_i(t) - d_i(t) & \text{for } 0 \leq t < \theta_i \end{cases} \quad (1)$$

$y_i(t)$ denotes the inventory level, $u_i(t)$ is the acquisition order with delay θ_i, and $d_i(t)$ the incoming demand. The function $\phi_i(t)$ describes the initial state of the system. In the next of the paper, and for the sake of simplicity, we shall consider that $\dot{y}_i(t) = 0$ for $0 \leq t < \theta_i$.

2.2 Constraints and Objective

The present part aims to define the controller parameters of each stage i, such as the end consumer demand $d_c(t)$ will be satisfied, and taking into account the different constraints of each single level i, $i = 1 \ldots, N$, as follows. The inventory levels are such that

$$y_i(t) \in [0, Y_{max_i}] \text{ , for } t \geq 0 \text{ ,} \quad (2)$$

The supplying order rates must verify

$$u_i(t) \in [0, U_{max_i}] \text{ , for } t \geq 0 \text{ .} \quad (3)$$

Y_{max_i} and U_{max_i} denote, respectively, the maximal capacities of storage and production of the stage i. For the retailer stage $i = 1$, the incoming demand is the end customer demand $d_c(t)$. It verifies the same assumption, namely

$$d_c(t) \in [0, D_{max}] \quad (4)$$

with D_{max} being the maximal customer demand specified to the considered supply chain. The additional constraints arising from the network structure are about the incoming demand of each level, where $d_i(t) = u_{i-1}(t)$ for $i = 2, \ldots, N$, such that

$$d_i(t) \in \left[0, U_{max_{i-1}}\right] \text{ .} \quad (5)$$

The objective of this study consists first to define the control law $u_i(t)$ at each level, in order to fulfill on line the consumer demands $d_c(t)$, and replenishing the inventory to a referential target denoted y_{c_i}. This control law aims to stabilize the delayed system (1) while ensuring the fulfillment of the constraints (2) and (3), for every variable and bounded end-consumer demand verifying the assumption (4). We analyze then impact of the proposed inventory control on the supply chain performances. In order to facilitate the understanding, we consider initially the inventory control problem for single logistic system composed of a basic logistic system in the section 3. Two cases are then considered (i) the lead time is well defined; (ii) the lead time is defined with an uncertainty interval. Then, we develop the proposed approach for a multi-sages supply line in the section 4. In each model, we describe the inventory control structure and give the necessary and sufficient conditions for the controller design.

3. INVENTORY CONTROL FOR SINGLE SYSTEM

3.1 Case Study

We consider a single logistic system wherein $y(t)$ is the inventory level, $u(t)$ the production rate with a delay θ, and $d_c(t)$ the customer demand rate. The inventory level dynamics is then described by the following equation.

$$\dot{y}(t) = u(t-\theta) - d_c(t) \text{ , for } t \geq \theta. \quad (6)$$

The customer demand $d_c(t)$ is considered unknown in advance but assumed to be upper bounded by an amount denoted D_{max}. As already mentioned, supplying unit and inventory are limited resources, which are formulated as follows

$$y(t) \in [0, Y_{max}] \text{ , for } t \geq 0 \text{ ,} \quad (7)$$

and for the supplying rate as follows

$$u(t) \in [0, U_{max}] \text{ , for } t \geq 0 \text{ .} \quad (8)$$

The assumption on the consumer demand is formulated such as

$$d_c(t) \in [0, D_{max}] \text{ , for } t \geq 0 \text{ .} \quad (9)$$

The controller design task consists of defining a controller which will stabilize the delayed system (6) while ensuring the fulfillment of the constraints (7) and (8), for every unknown and bounded customer demand verifying the assumption (9). Two cases are then considered: (i) the lead time is well defined; (ii) the lead time is defined with an uncertainty interval.

3.2 Case of Exact Lead Time θ

- *Inventory Control Structure*

Regarding to the nature of the system, we apply *a saturated command law based on a feedback predictor structure* such that

$$u(t) = \underset{[0, U_{max}]}{sat} [K(y_c - z(t))] \text{ ,for } t \geq 0. \quad (10)$$

y_c is the inventory target of $y(t)$ and K the controller gain. sat is a saturation function which introduces non-linearities in the closed-loop scheme. $z(t)$ is the prediction of the future state of the system, that corresponds to the inventory level at $(t+\theta)$. When the lead time θ is well defined, the control law approach for this case is developed in (Abbou et al., 2013). We recall here the main results. Using the feedback-predictor structure, also known as model reduction or Arstein's reduction (Artstein (1982)), the basic idea of state prediction is to compensate the time delay θ by generating a control law that enables one to directly use the corresponding delay-free system, thanks to the prediction expressed by

$$z(t) = y(t+\theta) + \int_t^{t+\theta} d_c(\tau) d\tau \text{ , for } t \geq 0. \quad (11)$$

By time derivation of the equation (11), one can see that the resulting system

$$\dot{z}(t) = u(t) - d_c(t) \text{ for } t \geq 0 \text{ ,} \quad (12)$$

is delay-free. The system (12) is the reduced model of the system (6)-(10). In Artstein (1982), the author demonstrated that the control law $u(t)$ is admissible for the closed loop system (6)-(10) if and only if it is admissible for the system (12)-(10). Our approach is then based on the use of the reduced system (12) to obtain the conditions to design the controller. Those conditions are summarized in the theorem 1.