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IFAC-PapersOnLine 48-3 (2015) 448–453

A Column Generation Based Heuristic for the Capacitated Vehicle Routing Problem with Three-dimensional Loading Constraints

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Abstract: This paper addresses an integrated problem of routing and loading known as the three-dimensional loading capacitated vehicle routing problem (3L-CVRP). 3L-CVRP consists of finding feasible routes with minimum total travel cost while satisfying customers' demands expressed in terms of cuboid and weighted items. Practical constraints related to connectivity, stability, fragility, and LIFO are considered as parts of the problem. 3L-CVRP is addressed by using a column generation (CG) technique based heuristic. To generate new columns, an integrated approach using the shortest path problem and 3D loading problem is applied. To speed up the CG technique, fast CG is also carried out by applying a heuristic pricing method. The CG technique outperforms the efficient tabu search technique proposed in the literature in terms of solution quality and execution time.

Keywords: three-dimensional loading problem, routing problem, column generation

1. 1. INTRODUCTION

Most vehicle routing problems consider only those capacity constraints which require that the total weight of products transported by a vehicle should not exceed its capacity (Cordeau et al., 2002). While in real world distribution systems, the shape of the products is also a key factor. Therefore, vehicle routing problems must take into account loading constraints that identify feasible placements of products within vehicles based on their shapes and properties.

In the literature, vehicle routing problems with loading constraints are classified into two types, two-dimensional and three-dimensional loading capacitated vehicle routing problems (2L-CVRP and 3L-CVRP respectively). In 2L-CVRP, the customer's demands are expressed in terms of distinct rectangular items. Items cannot be stacked on top of each other due to their fragility, weight, or large dimensions. One of the applications of this problem is the distribution of kitchen appliances such as refrigerators.

Iori et al. (2007) were the first to propose an exact approach to solve 2L-CVRP. In their approach, routing costs are minimized by a branch-and-cut algorithm, and loading aspects are iteratively checked by a branch-and-bound algorithm. The authors tested their approach on benchmark instances derived from the classical CVRP test problems involving up to 35 customers and more than 100 items. Gendreau et al. (2006) solved larger 2L-CVRP instances with up to 255 customers and 786 items by employing a tabu search (TS) method, where customers are relocated through a generalized insertion procedure (GENI) (Gendreau et al., 1992). Authors also applied a two-dimensional strip packing problem in order to guarantee the optimal placement of the load components. Interested reader may refer to Fuellerer et al. (2009), Zachariadis et al. (2009), Duhamel et al. (2011) for other methods for 2L-CVRP.

In 3L-CVRP, the demands of customers are expressed in terms of cuboid and weighted items. The aim of the problem is to find feasible routes with the minimum total traveling cost while satisfying customers' demands and practical loading constraints. 3L-CVRP, as one of the rich routing problems, draws a great deal of attention in supply chain management (Schmid et al., 2013). In particular, this problem has significance for applications that deal with many large items and in which the loading requirement is not trivial. Some examples include distribution of household appliances and mechanical components.

There is no exact algorithm for 3L-CVRP in the literature. Most researchers simply extend metaheuristic algorithms from 2L-CVRP to apply to 3L-CVRP. As mentioned by Vidal et al. (2013), most existing techniques for 3L-CVRP are based on TS combined with efficient packing heuristics. Gendreau et al. (2006) were the first to employ TS to tackle routing aspects of the problem and applied two packing heuristics to handle loading constraints. They evaluated their algorithm based on vehicle routing instances adapted from the literature as well as on new real-world instances. Tarantilis et al. (2009) described a hybrid metaheuristic methodology called GTS that combines the approaches of TS

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.06.122

and guided local search (GLS). They showed that GLS improves the solution attained by TS within a variable neighborhood search. The loading characteristics in this method were determined by employing a collection of packing heuristics. GTS improves average solution of TS of Gendreau et al. (2006) by 3.54%.

Fuellerer et al. (2010) solved 3L-CVRP by means of a highly efficient ant colony optimization (ACO) algorithm, adapted of the savings-based ants (Reimann et al., 2004). They handled routing aspects by an ant-based procedure. To deal with loading components, the ACO employs and iteratively invokes the local search and packing heuristics used in Gendreau et al. (2006). The ACO outperforms TS of Gendreau et al. (2006) in 26 out of 27 instances and GTS of Tarantilis et al. (2009) in 23 out of 27 instances. We refer the reader to (Wang et al., 2009) and (Iori and Martello, 2010) for a survey of 3L-CVRP.

In this paper we propose a column generation (CG) technique based heuristic to solve strong NP-hard 3L-CVRP. First 3L-CVRP is formulated as a set-partitioning model based on Dantzig-Wolfe decomposition. This model is then split into a master problem that is a linear relaxation of model and a sub-problem. Using CG technique, we in fact obtain a solution to master problem. If such solution is integer, then it would be a solution to the set-partitioning formulation. Sub-problem is used to discover new columns.

The valid columns are generated by applying two different methods. In the first method, an elementary shortest path problem is solved to obtain routes with negative reduced cost. Then an extreme point-based efficient heuristic is employed in order to verify the feasibility of obtained routes in terms of loading constraints. In the second method, an efficient heuristic pricing is applied to attain feasible routes in terms of loading constraints with negative reduced cost. If there is no improvement in the master problem solution then the column generation stops. After terminating CG technique, a branching rule in a heuristic framework is applied in case the solution value of the master problem is non-integer.

The main contribution of this paper is that using a CG based heuristic technique; we produce good results on benchmark instances. CG technique outperforms the tabu search(TS) from Gendreau et al. (2006) and Guided Tabu Search (GTS) from Tarantilis et al. (2009) in terms of solution quality and computation time.

The remainder of this paper is organized as follows: A description of the 3L-CVRP and corresponding set-partitioning formulation are presented in next section. In Section 3, CG technique including pricing problem is detailed. Computational results are presented and analysed in Section 4. Finally, conclusion and suggestions for future work are given in Section 5.

2. PROBLEM DISCRIPTION

Let us consider a complete graph G = (V, E) where V = {0, *1,…, n*} is the vertex set and $E = \{(i,j): i,j \in V, i \neq j\}$ is the set of edges. Vertex 0 corresponds to the central depot and the other vertices correspond to the customers. A cost c_{ij} is associated with each edge *(i, j)* that represents traveling cost from vertex *i* to vertex *j*. Assume that a fleet of *m* identical vehicles is available in the central depot. Each vehicle has a weight capacity *D* and a three-dimensional loading space of length *L*, width *W*, and height *H*. The demand of each customer *i (i = 1,…, n)* is expressed in terms of a set of cuboid items CI_i with a total volume s_i and a total weight capacity d_i. It is assumed that $s_i \leq L.W.H$ and $d_i \leq D$. Each item $I_{ik} \in CI_i$ *(k = 1, 2 . . . m$_i$)* is characterized by the length l_{ik}, width w_{ik}, height h_{ik}, and fragility status f_{ik} ($f_{ik} = 1$ for fragile items and 0 for non-fragile ones).

The aim of the 3L-CVRP is to identify a set of vehicle routes with the minimum total traveling cost while satisfying the following constraints:

- The number of vehicle routes selected in the solution must be less than or equal to the number of vehicles available in the depot;
- Each vehicle route must start and end at the depot;
- Each customer must be served by exactly one vehicle and visited only once;
- The total weight capacity and volume of the items placed in each vehicle must not exceed the weight capacity and volume of the vehicle; and
- All items must be orthogonally packed into the vehicles without overlapping, while satisfying rotation, stability, fragility, and last in first out (LIFO) constraints defined below.

Rotation constraints demand that items should be loaded with a fixed vertical orientation, that is, they can be rotated only by 90 degrees on the horizontal plane. In the fragility constraints, only fragile items can be stacked on other fragile items; any items can be stacked on the non-fragile items. To meet stability constraints, each item that is not packed directly on the vehicle floor should be stable in the vehicle and supported by a sufficient surface comprising other items. The supporting surface of an item should be at least 75% of the base area of the item. LIFO constraints require that all items packed into a vehicle should be directly unloaded through a sequence of straight movements parallel to the length of vehicle without repositioning other items. When a customer is visited, its items should not be blocked by or stacked under items that will be delivered to customers later on the route.

2.1 Set-partitioning Formulation

We express 3L-CVRP as set-partitioning formulation (Dantzig-Wolfe decomposition) in order to use CG technique. The key idea of set-partitioning formulation for the 3L-CVRP is to enumerate all feasible routes of the problem. A feasible route is defined as a vehicle trip that starts and ends at the depot, while visiting a subset of customers and satisfying capacity and loading constraints.