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Advanced Monte Carlo Method for model uncertainty propagation in risk assessment

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Abstract: In this paper, an Advanced Monte Carlo Method based on interval analysis approach and Monte Carlo simulation is proposed in order to propagate uncertainties in an atmospheric dispersion model. The purpose is to compute with accuracy the geographical region in which the concentration of the considered toxic gas is less than the threshold of irreversible effects. The problem of uncertainty propagation is tackled in order to assess the risk at the event of an accident, which may have an important impact on population. The estimation of gas concentration is based on an effect model associated with the studied dangerous phenomenon where some model inputs are known with imprecision. The principle of the proposed method is to generate random interval supports of model inputs instead of random values in order to increase accuracy and reduce the sampling size. The Advanced Monte Carlo Method is applied and compared for estimating uncertainty on the computed region with the classical Monte Carlo simulation.

Keywords: Risk assessment, atmospheric dispersion, uncertainty propagation, interval analysis, Monte Carlo.

1. INTRODUCTION

The assessment of technological risks is a decision aid that aims to rank or quantify risks to human in order to prioritize management actions and the allocation of resources. Industrial plants or manufacturing systems may stock, produce, transform and transport dangerous goods. In case of an accidental event, the risk intensity has to be evaluated, especially near heavily populated areas. The quantitative risk evaluation is made in using an effect model able to quantify risk intensity.

In this paper, accidental releases of hazardous gases are considered. So risk intensity is related to the concentration of the released toxic gas. In this way, an atmospheric dispersion model is used to estimate this concentration at a given geographical position. Dispersion models can be classified in three classes as follows : Gaussian models, integral type models and 3D or computational fluid dynamics (CFD) models. They can be used in the form of analytical expressions or computing programs. This dispersion model includes inputs such as source term, weather conditions, model parameters which may be measured, estimated or deduced with uncertainty (Oberkampf and Alvin (2002); Pulkkinen and Huovinen (1996)). These uncertain inputs lead to some uncertainty on the estimated gas concentration, and so on the computed dangerous area where gas concentration should exceed regulatory thresholds.

In recent years, several approaches have been developed in several areas in order to study and quantify the effects of uncertainties on the manipulated data like fuzzy sets approach(Zadeh., 1978), probabilistic approach (Robert and Casella, 1999) and set membership approach (Moore, 1960), in other word for estimating the propagation of uncertainties on the model output. The uncertainty propagation is equivalent to calculate a confidence interval which delimits the output variation between two lower and upper limits deduced from uncertain inputs.

In this work an Advanced Monte Carlo Method (AMCM) is proposed for estimating the uncertainty propagation based on two techniques. The first one is a probabilistic approach (Monte Carlo) and the second one is a set membership approach based on interval analysis. They aim at representing an uncertain input respectively by a random variable following a given probability distribution or a variable defined by a bounded support. The principle of the proposed method is to generate random interval supports of model inputs instead of random values in order to increase accuracy and reduce the sampling size. The obtained results of model uncertainty propagation by means of the proposed method are compared here with the results given by Monte Carlo simulation and interval analysis method in order to study the variability of uncertainty propagation for the three approaches. Then the inversion problem of the effect model using the MCS and AMCM approaches for taking into account model uncertainties is treated in order to determine the geographical area in which gas concentration is less than the threshold of irreversible effects.

The organization of this paper is as follows. In the next section the uncertainty propagation approaches are presented. In section 3, the Advanced Monte Carlo Method is explained. The results of uncertainty propagation obtained with the different approaches are reported and compared in section 4. Then the dangerous areas computed with MCS and AMCM are detailed in section 5. Finally, the conclusion is presented in the last section.

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2. UNCERTAINTY PROPAGATION APPROACHES

2.1 Monte Carlo approach for uncertainty propagation

Monte Carlo Simulation (MCS) is a computational and probabilistic method that can be used to propagate the uncertainty coming from inputs to the model output. It is a less complex method relative to the analytical methods, but it requires much more computing resources (Morgan and Henrion, 1990; Gentle, 2003; Glasserman, 2004; Ayyub and Klir, 2006). In the following, the analytical model of atmospheric dispersion will be written in the form of a mathematical relation (1) describing the studied dangerous phenomenon:

$$y = f(x_1, ..., x_p) \tag{1}$$

Where y and x_i denote respectively the gas concentration and the i^{th} scalar model input (wind speed, conditions emission point, release flow...) influencing the model output.

Monte Carlo simulation process :

(1) Define the output and the input factors of the mathematical model
(2) Associate a probability density for each model input on which a MCS is performed.
(3) Generate a N-sample $X_{(N)}$ of size p, where N is the number of simulations and p is the number of model inputs.
(4) Calculate the resulting model value of y for each independent sample of size p.
(5) Perform the propagation of uncertainties on the model output by using these N values of y.

Implementation of the Monte Carlo simulation

Figure 1 presents the calculation phase of uncertainty propagation using MCS.

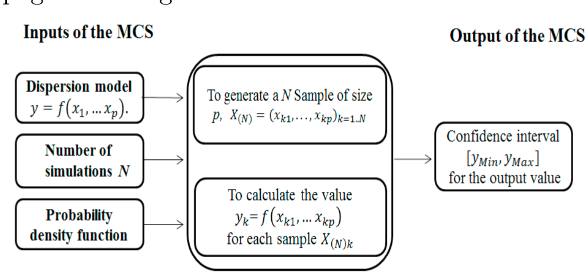

Fig. 1. The calculation phase of uncertainty propagation.

Three main steps are executed during the implementation of the Monte Carlo simulation : generation of N samples of p inputs according to probability density functions, evaluation of the model output for each sample and finally estimation of the model output and the associated uncertainty. The final result of uncertainty propagation is the confidence interval for the model output. Based from the N values $y_1, y_2, ..., y_N$, the uncertainty u is defined as:

$$u = \frac{y_{Max} - y_{Min}}{2} * \frac{100}{y_{NominalValue}}. \tag{2}$$

Where $y_{NominalValue}$ is the output value of the model without uncertainty on the model inputs. The y_{Min} and y_{Max} define respectively the minimal and maximal values of $y_{k,k=1,...,N}$.

2.2 Interval analysis approach for uncertainty propagation

Uncertainties may be also represented by intervals around a nominal value. Interval modeling consists in describing an uncertain input by an unknown bounded variable, whose known support defines its feasible value set.

Interval Arithmetic

By definition, an interval is a closed and bounded set of real numbers (Moore., 1979),(Neumaier, 1990).
If x denotes a bounded real variable, then the interval $[x]$ which it belongs is defined by:

$$[x] = \{x \in \mathbb{R} | x^- \leq x \leq x^+\} \tag{3}$$

The real numbers x^- and x^+ are respectively the lower and upper limits of $[x]$. In general, the range $[x]$ is denoted as follows: $[x^-, x^+]$. The operation result on finite intervals is defined by two bounds which are obtained by working only on the bounds of these intervals. In this way, interval arithmetic is an extension of real arithmetic. For a real arithmetic operation $\circ \in \{+, -, *, /\}$, the corresponding interval operation on intervals $[x]$ and $[y]$ is defined by:

$$[x] \circ [y] = \{x \circ y | x \in [x], y \in [y]\}. \tag{4}$$

Interval arithmetic considers the whole range of possible instances represented by an interval model. In the classic set-theory interval analysis, given a $\mathbb{R}^p$ to $\mathbb{R}$ continuous function $y = f(x_1, ..., x_p)$, the interval united extension $[f]$ of f corresponds to the range of f-values on its interval argument $([x_1], ..., [x_p])$ in $I(\mathbb{R}^p)$:
$[f]([x_1], ..., [x_p]) = \{f(x_1, ..., x_p) | x_1 \in [x_1], ..., x_p \in [x_p]\} = [min\{f(x_1, ..., x_p) | x_i \in [x_i]\}, max\{f(x_1, ..., x_p) | x_i \in [x_i]\}]$ $i = 1, ..., p$.
This notion can be extended to a vector $\mathbf{x}$ composed of p bounded real variables x_i,$i = 1, ..., p$. In this case, the support of $\mathbf{x}$ becomes an interval vector also called a box $[\mathbf{x}]$: $[\mathbf{x}] = [[x_1], ...[x_p]]^T$. In order to introduce interval calculus, the most elementary principle is to evaluate the image of a box through a function f, i.e. to compute the value set: $S_f = \{f(\mathbf{x}) : \forall \mathbf{x} \in [\mathbf{x}]\}$. The result of the interval evaluation of $[f]([\mathbf{x}])$ leads to an overestimated interval containing S_f.

Pessimism

The interval calculation essentially suffers from a problem of pessimism, i.e it may lead to a computed interval which represents an overestimation of the sought value set. Indeed, the interval result after a series of mathematical operations is not necessarily minimal, so that an interval with a long width may be obtained. This problem is mainly due to the dependence phenomenon (Raissi (2004)). Dependency between bounded variables $x_{i,i=1,...,p}$ cannot always be taken into account when their interval supports are manipulated. In return, the advantage is that interval calculation is guarenteed in the sens that all situations are taken into consideration.
For example, let $[x] = [-1, 1]$, then $[x] - [x] = [-1, 1] - [-1, 1] = [-2, 2] \neq \{0\}$, the interval operation overestimates the exact domain $\{0\}$. In a general manner, pessimism depends on the occurrence of interval variables in the expression of $[f]$. It also depends on the widths of the manipulated intervals, indeed to work on smaller intervals reduces the pessimism phenomenon and increases accuracy of the uncertainty propagation.