Measurement 128 (2018) 63–70

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Experimental measurement of mesh stiffness by laser displacement sensor technique

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ARTICLE INFO

Keywords:
Laser displacement sensor
Spur gear
Gear Mesh stiffness
Cracked tooth

ABSTRACT

Accurate measurement of mesh stiffness of a given gear pair is important for understanding the dynamics of gearboxes. Photoelasticity and strain gauge techniques are the only experimental techniques that are suggested to measure the mesh stiffness of cracked spur gear pairs in the literature. In the above two experimental techniques, the gear body deformation was not taken into account for the mesh stiffness measurement. In this work, a new experimental technique is designed to measure the mesh stiffness. For this, a well-established laser displacement sensor technique (LDST) is used. In this method spur gear tooth deflection is measured along the line of action by using the laser displacement sensor, consequently, gear mesh stiffness is calculated. The experiment is also performed on cracked tooth pair to measure the mesh stiffness. The main advantage of this experimental technique is to measure the total deflection of gear tooth with gear body deformation. The investigation shows that the gear mesh stiffness is reduced when crack size is increased. For validating the results of the experiment, FEA (finite element analysis) is performed to estimate the mesh stiffness. The mean mesh stiffness is measured as 0.76×10^4 N/mm by LDST and 0.96×10^4 N/mm by FEA for the healthy case. The results of the experiment are found having a good match with that obtained from FE method.

1. Introduction

Gearboxes are generally used in aerospace, automobiles and industries. Condition monitoring and fault diagnosis of machines based on vibration analysis is the most effective method for predicting gear failure in advance. The mesh stiffness of a given gear pair is a key parameter of gearbox vibrations. An accurate measurement of mesh stiffness gives the correct dynamic behaviour of the gearbox. There are a number of factors which affect the mesh stiffness such as gear tooth geometry, contact ratio, pressure angle, rim thickness etc. [1–5]. The mesh stiffness is also affected by gear tooth faults such as crack, spalling, tooth breakage etc. [6–12]. Some investigations on mesh stiffness measurement for healthy and different gear tooth faults by different methods are discussed in the below paragraph.

Yang and Lin [13] presented computer-aided and an analytical study on the dynamics of meshing gear pairs. They considered axial compression, bending deflection, and coulomb friction. Kahraman and Singh [14] investigated the interactions between the mesh stiffness variation and gear backlash. Howard et al. [6] used FEA (finite element analysis) to know the effect of variation in the torsional mesh stiffness and used in their simplified gear dynamic model. Tian [15] included the shear deflection in the model of Yang and Lin [13] to evaluate the equivalent gear mesh stiffness and also showed the result of mesh stiffness for cracked tooth in his Master's Thesis. Parey et al. [16] used a six DOF (degree of freedom) gear model and included the effect of mesh stiffness in his study, which is calculated by the empirical method. Zouari et al. [17] presented a three-dimensional model to study the mesh stiffness of cracked tooth by FEA. Wu et al. [18] showed crack growth effect on mesh stiffness and response of gearbox. Chaari et al. [12] showed tooth breakage and spalling effect on the mesh stiffness. Chaari et al. [19] presented an analytical method (AM) for estimating the mesh stiffness and also applied the method for cracked tooth. Chen and Shao [11] studied the crack propagation in spur gear tooth and they considered the crack along the tooth width. Yang et al. [20] studied an anti-backlash gears to show the effect of friction and damping on mesh stiffness. Chen and Shao [21] investigated the gear mesh stiffness for root crack and also for tooth profile modification. Pandya and Parey [3] analysed the mesh results for different gear parameters. Pandya and Parey [5] investigated mesh stiffness of high contact ratio with cracks. Pandya and Parey [22] also improved mesh stiffness evaluation accuracy for contact ratios and crack path. Chen and Shao [4] included gear rim deformation in mesh stiffness evaluation. Guo and Parker [23] investigated analytically back-side contact gear mesh stiffness. Mohammed et al. [9] used a method to estimate the gear mesh

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https://doi.org/10.1016/j.measurement.2018.06.035
Received 11 May 2017; Received in revised form 11 May 2018; Accepted 19 June 2018

stiffness for large crack sizes. Pandya and Parey [24] proposed a photoelasticity method to investigate the cracked tooth mesh stiffness. Wan et al. [7] presented an algorithm to improve the mesh stiffness calculation with tooth crack. Liang et al. [8] suggested a method to estimate the gear mesh stiffness by modifying the cantilever beam model. In which they derived an equations of shear stiffness, bending stiffness, and axial compressive stiffness. This method is also used for cracks. Ma et al. [25] enhanced a gear mesh stiffness model for healthy pairs and for the base circle and gear root circle misalignment. Based on improved model crack paths are analysed. Saxena et al. [26] studied variation in mesh stiffness with shaft misalignment and friction force. Ma et al. [27] summarised different methods of mesh stiffness calculations in the review study. Ma et al. [28] investigated the effect on mesh stiffness for extended tooth contact. Raghuwanshi and Parey [29] used photoelasticity method to evaluate the gear mesh stiffness for full mesh cycle. Deng et al. [2] investigated the mesh stiffness of modified and unmodified asymmetric gears. Cui et al. [30] applied universal gear profile equation to calculate the mesh stiffness. Li et al. [31] predicted the contact effect on gear mesh stiffness. Chen et al. [32] compared the dynamic response between rectangular gear mesh stiffness and approximate mesh stiffness form. Ma et al. [1] improved analytical method of mesh stiffness and studied the mesh stiffness for tooth tip relief. Yu et al. [10] and Li et al. [33] studied the influences of spatial crack on mesh stiffness. The mesh stiffness evaluation are done by many researchers for healthy as well as faulty gears [34–38].

From the above literature, it is seen that the researchers are paying attention to improving the methods for estimating the mesh stiffness. The improvement in analytical models is the primary objectives of the researchers. However, only few experimental techniques viz. Photoelasticity technique [24,29] and strain gauge method [39] have been found in the literature to investigate the mesh stiffness for healthy and cracked tooth cases.

In these two experimental methods, the gear body deformation is not considered during gear mesh calculations. Instead of not considering gear body deformation these techniques require huge analytical expressions to evaluate the stiffness. For measuring the stiffness of cracked tooth, SIF (stress intensity factor) is also needed to evaluate the cracked tooth stiffness. The calculation of SIF is a very complicated and time-consuming exercise. To overcome these problems and assumptions an experimental technique based on laser displacement sensor (LDS) is proposed for investigating the mesh stiffness. The position of LDS on gear tooth has been suggested to measure the deflection accurately. The proposed methodology is explained in Section 2.

2. Methodology

The methodology proposed to estimate the mesh stiffness of healthy and cracked tooth case is shown by flow chart as given in Fig. 1. In this experimental method, the deflection is measured along the LOA (line of action) of a gear pair. The position of LDS is an important to measure the deflection in the direction of force. The LDS should be placed in such a way that the laser beam should coincide with the LOA of a given gear pair. To understand the position of LDS, see the Fig. 5. The factors on which the position of LDS depends are that how far the reflector is placed from the LDS and the laser beam should be perpendicular to the reflector. The distance between LDS and reflector should be within the range as given in Section 3. The stiffness is calculated with the help of this deflection. Now the individual tooth stiffness of one pair and Hertzian contact stiffness are connected in series and calculated the equivalent mesh stiffness of first and second pair separately. The Hertzian contact stiffness is calculated analytically. The equivalent mesh stiffness of double tooth contact pair (DTCP) is then calculated by parallel connection of equivalent mesh stiffness for two individual pairs as explained above.

The measurement methodology of cracked tooth (faulty tooth) mesh stiffness is also same as the healthy gear pairs. But the cracked gear tooth is analysed in one pair and another pair is kept healthy. The detailed calculation of mesh stiffness is explained in Section 2.1.

2.1. Mesh stiffness measurement

Single tooth stiffness can be written as the ratio of force to the deflection. The deflection and force should be along the LOA. In the case of gear pairs, the total equivalent stiffness of teeth which are in the mesh is termed as mesh stiffness. During the single tooth contact pair (STCP), the gear mesh stiffness is the series combination of individual tooth stiffness. In the experiment, the deflections are measured at the centre line of the tooth and calculated the stiffness. The contact deflection is not considered in the experiment. The contact stiffness viz. Hertzian stiffness is calculated analytically. Hertzian stiffness is added in the series combination with the stiffness obtained from experiments. For STCP, the force is equal on each tooth of pair and deflections can be different. So the mesh stiffness k_m of STCP can be expressed as [24,29,39];

$$k_m = \frac{1}{\frac{1}{k_1} + \frac{1}{k_h} + \frac{1}{k_2}} \tag{1}$$

where k_1 and k_2 are the tooth stiffness of meshed tooth of gear 1 and 2 respectively and k_h is the Hertzian contact stiffness.

The Hertzian contact stiffness can be written as [24,29,39];

$$k_h = \frac{EB\pi}{4(1-\upsilon^2)} \tag{2}$$

where υ is the Poisson's ratio, B is the face width of the gear tooth and E is the modulus of elasticity.

The individual tooth stiffness can be calculated as [19,24,39];

$$k_1 = \frac{F}{\delta_1} \tag{3}$$

$$k_2 = \frac{F}{\delta_2} \tag{4}$$

where δ_1 and δ_2 are the deflections of gear 1 and 2 respectively along the LOA and F is the total force.

For calculating the mesh stiffness for DTCP, the individual pair mesh stiffness is calculated with help of above Eq. (1) but the force is calculated by using load sharing factor.

The mesh stiffness of DTCP is the summation of two individual pair mesh stiffness and is given as [24,29,39];

$$k_m = k_{mf} + k_{ms} \tag{5}$$

where k_{mf} and k_{ms} are the first and second pair mesh stiffness respectively.

In cracked tooth case, the same procedure is applied for cracked tooth case mesh stiffness evaluation of cracked tooth pair. The tooth stiffness formula in Eq. (4) is given for stiffness of tooth of the gear 2. In the present experiment, crack was inserted in the tooth of gear 2. So the Eq. (4) is also used to evaluate the cracked tooth stiffness instead of the healthy tooth.

3. Experimental description

An experimental setup with complete accessories is shown in Fig. 2(a). The whole experiment consists of two units, one is the gear pair arrangement with loading and second one is the LDS unit. The LDS unit components are LDS, data acquisition, power supply and computer system. The gear pair arrangement components are supporting frame with gear 1 and gear 2 meshing, lever and load as shown in Fig. 2 (b). In the experiment, gear 2 is fixed with the help of locking nuts. One plate with a radial slot is provided behind the gear 2 so that we can fix the gear 2 at desired angle of rotation. Now gear 1 is in mesh with gear 2 and free to rotate on its axis. The torque is applied to the gear 1 with the