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Stability condition of discrete-time linear Hamiltonian systems with time-varying delay feedback interconnection ⋆

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Abstract: This paper is concerned with stability analysis of delay feedback structures within discrete port-Hamiltonian framework. We introduce a discrete dynamics that approximates linear port-Hamiltonian systems and is passive relatively to the same storage and dissipation functions. Stability of interconnected discrete systems is then addressed when considering time-varying delay feedback interconnection structure. A delay bounds-dependent stability condition is derived for variable and bounded delayed interconnection, reducing to a delay-independent condition for constant delay. A sufficient condition is formulated in terms of a feasibility problem under Linear Matrix Inequality (LMI) constraints. It is noticeable that the LMI parameters linearly depend on the network characteristics (damping and input matrices). Moreover, computing and storing past history of the discrete flow is no longer required. A numerical example illustrates the feasibility of the approach.

1. INTRODUCTION

Port-Hamiltonian systems (PHSs) [Maschke and van der Schaft (1992)] form a class of passive systems known to be composable: a network of PHSs belongs to the class. As stability follows from passivity, composability turns out to be an essential property since the network inherits passivity from its components. To set out stability issues including delays in this context, simply note that composability is derived from the network's topology which is characterized by a power-conserving interconnection structure, and that delay interconnection structures are no longer power-conserving.

Basically, the port-Hamiltonian framework draws a complex system as an oriented graph of energy exchanges (supported by edges) between subcomponents (associated with nodes). This energy-based description of physical systems is met in many engineering fields (as in mechanics, electronics, robotics and haptics). The topology of the graph (the set of energy links) defines the interconnection structure. Communication delays in this framework (encountered for instance in telemanipulation) are thus encoded by a delayed interconnection structure. We shall consider time-delay systems (TDSs) where the delays precisely and only occur in this structure.

In closed-loop schemes, it is well-known that delays may induce oscillations and instabilities [Niculescu (2001); Normey-Rico and Camacho (2007)]. Stability analysis is classically tackled by using Lyapounov-Krasovskii (LK) functionals to derive sufficient conditions in terms of a feasibility problem under linear matrix inequalities (LMI) constraints. Generally speaking, stability of TDSs is addressed from a state-space representation gathering the present and the past history of the trajectory. Considering linear discrete-time systems with time-varying (upper-bounded) delay, numerically tractable conditions are carried out based on the generator of the discrete flow using delay dependent LK functionals [Hetel et al. (2008)]. Note that the LMI grows as the upper bound of the delays does. We shall see that, with the class of linear discrete-time systems considered here, the size of the LMI does not depend anymore on the upper bound: the LMI criterion is directly derived from the network characteristics.

Regarding continuous-time delay interconnected nonlinear PHS, stability analysis has been investigated by LK functionals and Jensen's inequality in [Kao and Pasumarthy (2012)], and a less conservative criterion can be derived using improved Wirtinger-based inequality [Aoues et al. (2014)]. Both results propose a construction of LK functionals based on the subsystems energies and the network characteristics, leading to a LMI criterion.

In this paper, we consider the class of linear discrete-time port-controlled Hamiltonian systems with dissipation (LPCHD), that is a PHS with a quadratic energy and a damping matrix. Stability analysis of the feedback interconnection is achieved in terms of LMI condition. Delay bound-dependent (resp. -independent) condition is derived for variable and bounded (resp. constant) time-delay located in the interconnection structure. Opposite to the general approach (derived with delay-dependent LK), the criterion is derived from the network damping and input matrices which is noticeable from a dimensionality point of view: the computational cost becomes independent from the

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size of the delay variation. This result extends the ideal case (meaning without delay) processed in [Aoues et al. (2013)].

Content is as follows. Section 2 introduces the linear discrete-time LPCHD systems and states the stability issue. The main result is presented in Section 3: LK functional is proposed and LMI stability conditions are derived. Section 4 compares the result with the classical approach from dimensionality viewpoint. A numerical example is given in Section 5. Conclusion and proofs in Appendix end the paper.

2. LINEAR PORT-CONTROLLED HAMILTONIAN SYSTEMS WITH DISSIPATION (LPCHD)

2.1 Continuous-time settings

Throughout the paper, we shall consider the class of *linear port-controlled Hamiltonian systems with dissipation* (LPCHD), which dynamics is described by the following equations [van der Schaft (1999)]

$$\Sigma(x) : \begin{cases} \dot{x}(t) = [J-R]\,Qx(t) + gu(t) \\ y(t) = g^T Qx(t) \end{cases} . \qquad (1)$$

$x \in \mathbb{R}^n$ is the state vector and $H(x) = \frac{1}{2}x^T Qx$ is the total energy with $Q = Q^T > 0$. The structure matrix J is a symplectic matrix (full rank and $J + J^T = 0$), and the damping matrix R satisfies $R = R^T \geqslant 0$. $g \in \mathbb{R}^{n\times m}$ is the input matrix. $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^m$ its conjugate port-output.

By integrating $\frac{\mathrm{d}}{\mathrm{d}t}H$ from t_0 to $t > t_0$, one gets the energy balance equation

$$H(t) - H(t_0) = \int_{t_0}^{t} y^T(s)u(s)\mathrm{d}s - \int_{t_0}^{t} x^T(s)Q^T RQx(s)\mathrm{d}s. \quad (2)$$

Equation (2) reflects that H is a storage function. Σ given by (1) is thus a *passive* system whenever H is bounded from below. Moreover when $R = 0$, Σ is said to be *lossless*.

Remark 2.1. Observe that equation (2) states $\frac{\mathrm{d}}{\mathrm{d}t}H = -x^T Q^T RQx$ in the unforced case. Then first when R is only symmetric and positive, the passivity equation translates Lyapunov stability. Second, when R is now positive definite, asymptotic stability follows.

Consider $\Sigma_1(x)$ and $\Sigma_2(z)$ given by (1) with constitutive elements $(J_i, R_i, Q_i, g_i)_{i=1,2}$. The feedback interconnection is characterized by the constraints on the port variables $u = -w$ and $v = y$ as drawn in Fig. 1.

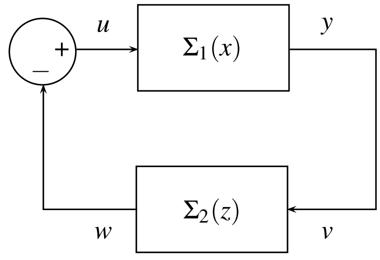

Fig. 1. Feedback interconnection structure

The resulting system, denoted by $\Sigma_{12}(X)$, is governed by the equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} J_1 - R_1 & -g_1 g_2^T \\ g_2 g_1^T & J_2 - R_2 \end{bmatrix} \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix}, \qquad (3)$$

or equivalently

$$\dot{X} = [J_{12} - R_{12}]Q_{12}X,$$

where $X = \begin{bmatrix} x^T & z^T \end{bmatrix}^T$, $J_{12} = \begin{bmatrix} J_1 & -g_1 g_2^T \\ g_2 g_1^T & J_2 \end{bmatrix}$, $R_{12} = \mathrm{diag}(R_1, R_2)$ and $Q_{12} = \mathrm{diag}(Q_1, Q_2)$.

Noting that the total energy H_{12} is given as the sum of subsystems energies $H_{12}(X) = H_1(x) + H_2(z) = \frac{1}{2}X^T Q_{12}X$, the structure matrix J_{12} gathers the skew-symmetric part and R_{12} the damping part, one concludes that Σ_{12} belongs to the class of LPCHD. The class invariance under interconnection is referred to as *composability*.

Therefore, thanks to composability, stability analysis of the interconnected system is conducted as previously: integrating $\frac{\mathrm{d}}{\mathrm{d}t}H_{12}$ leads to

$$H_{12}(X(t)) - H_{12}(X(t_0)) = -\int_{t_0}^{t} X(s)^T Q_{12}^T R_{12} Q_{12} X(s)\mathrm{d}s, \quad (4)$$

and one concludes as in Remark 2.1.

2.2 Discrete-time settings

Discrete-time approximation of Hamiltonian systems can not be performed using arbitrary numerical schemes. Indeed, the *unforced* case deserves dedicated schemes as energetic and geometric integrators [Feng and Qin (2002)], meaning that either the Hamiltonian or the volume is preserved. The *input-output* case remains an open issue, except for the linear case treated in [Greenhalgh et al. (2013)] from a dissipative viewpoint by a $\theta - \lambda$ method and in [Aoues et al. (2013)] from a lossless viewpoint by a midpoint scheme.

We introduce discrete-time LPCHD as follows.

Definition 2.1. A discrete-time LPCHD with state x_k is given by the set of equations

$$\Sigma(x_k) : \begin{cases} \dfrac{x_{k+1} - x_k}{\Delta t} = [J-R]\,Q\dfrac{x_{k+1} + x_k}{2} + gu_k \\ y_k = g^T Q\dfrac{x_{k+1} + x_k}{2} \end{cases}, \qquad (5)$$

where $Q = Q^T > 0$, $J = -J^T$ is full rank, $R = R^T \geqslant 0$ and $\Delta t > 0$. (u_k, y_k) are called discrete conjugate ports.

Proposition 1. $\Sigma(x_k)$ given by (5) is a discrete-time approximation of $\Sigma(x)$ given by (1).

Proposition 2. $\Sigma(x_k)$ is a passive system relatively to the storage function $H(x_k) = \frac{1}{2}x_k^T Qx_k$. Moreover, (5) encodes losslessness when $R = 0$.

Proof. The discrete energy balance $\Delta H_k := H_{k+1} - H_k$ along the trajectories of (5) writes

$$\begin{aligned} \Delta H_k &= \left\langle Q\frac{x_{k+1} + x_k}{2}, (x_{k+1} - x_k) \right\rangle \\ &= \Delta t \left\langle Q\frac{x_{k+1} + x_k}{2}, [J-R]\,Q\frac{x_{k+1} + x_k}{2} + gu_k \right\rangle \qquad (6) \\ &= \Delta t\, y_k^T u_k - \Delta t \left[Q\frac{x_{k+1} + x_k}{2} \right]^T R \left[Q\frac{x_{k+1} + x_k}{2} \right]. \end{aligned}$$

Equation (6) shows that $\Sigma(x_k)$ is a passive system relatively to the storage function H. It is the discrete counterpart of equation (2). The system is clearly lossless when $R = 0$. ■