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Port-Hamiltonian Modelling for Buckling Control of a Vertical Flexible Beam with Actuation at the Bottom

Megha V. Trivedi *, **Ravi N. Banavar** *, **Paul Kotyczka** **

* *Systems and Control Engineering, Indian Institute of Technology, Bombay, Mumbai 400076 (e-mail: {megha,banavar}@sc.iitb.ac.in)*
** *Institute of Automatic Control, Technische Universität München, 85748 Garching (e-mail: kotyczka@tum.de)*

Abstract: The use of beams and similar structural elements is finding increasing application in many areas, including micro and nanotechnology devices. For the purpose of buckling analysis and control, it is essential to account for nonlinear terms in the strains while modeling these flexible structures. Further, in modeling of micro and nanotechnology devices, the micro length scale parameter effects can be accounted by the use of a 2 dimensional stress-strain relationship. This paper studies the buckling effect for a slender, vertical beam with a tip-mass at one end and fixed on a movable platform at the other. For the purpose of illustration, the movable platform is assumed to be a cart. Accounting for a 2 dimensional stress-strain relationship, nonlinear expressions for strains, and incorporating an inextensibility constraint of the beam, the Hamiltonian equations of motion are obtained. The equations of motion are then cast in a port-Hamiltonian form with appropriately defined flows and efforts. We then carry out a preliminary modal analysis of the system to describe candidate post-buckling configurations and study the stability properties of these equilibria. The vertical configuration of the beam under the action of gravity is without loss of generality, since the objective is to model a potential field that determines the equilibria.

Keywords: Nonlinear beam model, distributed-parameter systems, port-Hamiltonian systems, boundary conditions, buckling.

1. INTRODUCTION

This paper presents port-Hamiltonian (PH) modelling to study the buckling effect for a slender, vertical Euler-Bernoulli (EB) beam with a tip-mass at one end and fixed on a movable platform (the actuator) at the other. For the purpose of illustration, the movable platform is assumed to be a cart.

The problem of buckling analysis and control of EB beam subjected to an axial load, has been dealt widely in the literature (see Meressi & Paden (1993), Thompson & Loughlan (1995), Wang (2010)). However, the EB beam model used there are small deflection models based on the assumption of small strains and rotation, and utilize the linear expressions of strains. To describe the large deflections in a beam (as expected in case of buckling), the nonlinear terms in the strain need to be considered. In Voß et al. (2008) the EB beam model has been derived using a nonlinear expression for the axial strain. However, the model thus obtained is an one dimensional model. The beam model based on modified couple stress theory that accounts for the nonlinear terms in the normal and shear strain, too seems to be more accurate (see Ma et al. (2008)), as it describes the beam bending problem in two dimensions rather than just restricting it to one dimension as done in conventional EB models. In modeling of micro and nanotechnology devices, the micro length scale parameter effects can be accounted by the use of a 2 dimensional stress-strain relationship. In the generalized beam theory described in Reddy & Mahaffey (2013), it is shown that by including the nonlinear terms in normal and shear strain, the micro length scale parameter effect can be accounted for in the beam model. This effect is significant in micro/nano beams. Further, for buckling analysis, it is essential to account for the inextensibility constraint of the beam (see Patil & Gandhi (2014)). The model derived in our paper, incorporates a 2 dimensional stress-strain relationship, non-linear expressions for the strains and the inextensibility constraint of the beam. The non-linear model thus obtained describes the large deflection of micro/nano beams and serves as an appropriate model for describing the buckling phenomena.

From the point of view of control, it is convenient to represent these mathematical models in a PH framework since there are well-established energy based control synthesis techniques for such models. There are several references discussing the control of flexible beams employing their PH model. The problem of stabilizing the displacement of an

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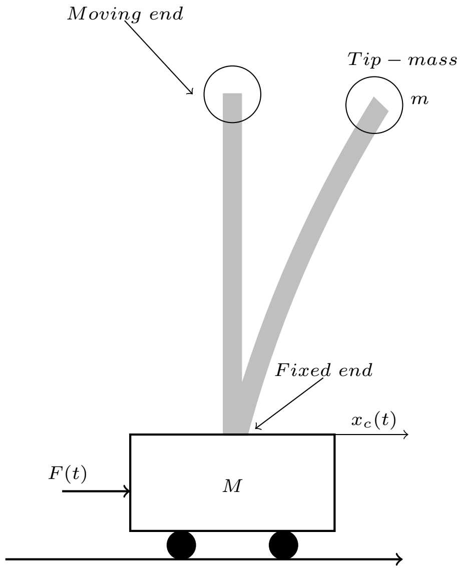

Fig. 1. Flexible beam with a tip-mass on a cart

EB beam using a boundary control approach is discussed in Osita et al. (2001). The approach of control by damping injection (both boundary control and distributed control) and control by interconnection and energy shaping of the linear Timoshenko beam can be found in Macchelli & Melchiorri (2004a) and Macchelli & Melchiorri (2004b). In Banavar & Dey (2010), a stabilizing controller has been obtained for a flexible beam fixed to a moving cart. However, the beam models discussed in these references are linear ones that describe small deflection or vibration control problems. Although, in Nishida & Yamakita (2005) the PH representation of flexible beams under large deformation has been obtained, the model does not account for the axial load, actuation nor the inextensibility constraint. Our work addresses the challenge of obtaining the PH representation of nonlinear beam model also accounting for the inextensibility constraint. The vertical configuration of the beam under the action of gravity is without loss of generality, since the objective is to model a potential field that determines the equilibria.

The paper is organized as follows: The system description and the nomenclature used in the paper is presented in Section II . In section III, the equations of motion are derived using the extended Hamilton's principle followed by the PH model of the cart, beam and the tip-mass in Section IV. A preliminary modal analysis of the system to describe candidate post-buckling configurations and study of the stability properties of these equilibria is discussed in Section V. Finally in Section VI, the conclusion are drawn and the future objective have been set.

2. SYSTEM DESCRIPTION

A schematic of the system of our interest is shown in Fig. 1. It comprises of the following subsystems: a moving cart, a flexible beam that is fixed at the bottom to the cart and has a rigid tip-mass attached to the upper free end.

The nomenclature used in the paper is as follows: the beam has a length L, width a and thickness b. The beam is symmetric with respect to the body frame (x_B, y_B, z_B) as shown in Fig. 2. Let $y \in [0, L]$ be the position of any

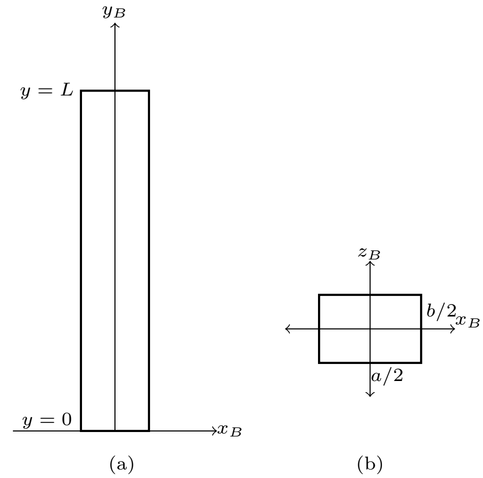

Fig. 2. (a) Undeflected profile and (b) cross section of the beam

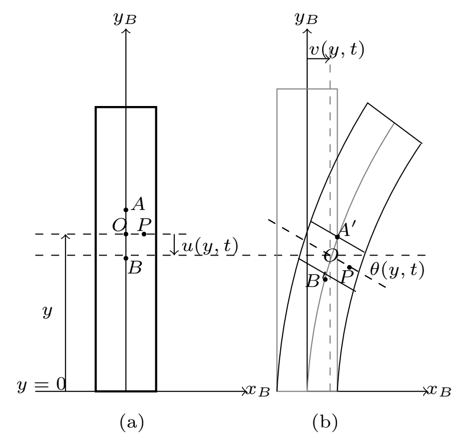

Fig. 3. (a) Undeflected profile of the beam (b) Beam under tranverse deflection

point O on the mid-plane, from the base of the beam. Let the horizontal deflection of the beam from its upright profile, the axial deflection due to the loading effect of the tip-mass, and the rotation of the cross-section of the beam due to bending at O be $v(y,t)$, $u(y,t)$, and $\theta(y,t)$ respectively (refer to Fig. 3). The tip-mass is attached to the flexible beam with the center at $y = L$. It has a radius r and mass m. The mass translates in the transverse and vertical direction. Let $v_m(t) = v(L,t)$, $u_m(t) = u(L,t)$ and $\theta_m(t) = \theta(L,t)$ represent the horizontal displacement, vertical displacement and rotation of the tip-mass from the vertical axis, respectively. The acceleration due to gravity is denoted by g. The cart has a mass M, displacement $x_c(t)$ with respect to the fixed frame of reference as shown in Fig. 5 and $F(t)$ is the actuation force applied to the cart.

3. EQUATIONS OF MOTION

In this section, we derive the generalized equations of motion for an EB beam using the simplified Green-Lagrange strain tensor. The derivation accounts for the micro length scale effects. The EB beam theory assumes that,