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On port-Hamiltonian modeling and control of quaternion systems

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Abstract: A quaternion representation is often used to describe the attitude of a rigid body type spacecraft since it does not have any singular point whereas the conventional Euler angle description intrinsically has one. However, the dynamical equation with quaternions become more complicated than those described by Euler angles. The scope of this paper is to provide a basis of modeling and control of those systems using port-Hamiltonian system formulation to remove some of those difficulties in control of those systems. A stabilization procedure based on passivity based control is proposed and a sufficient condition for artificial potential energy are derived for a class of simple systems with quaternions.

Keywords: Passivity based control, quaternions, aerospace systems

1. INTRODUCTION

Port-Hamiltonian systems are introduced as generalization of conventional Hamiltonian systems in order to describe physical systems with energy dissipation van der Schaft (2000). A stabilization method/strategy utilizing energy dissipation is called passivity based control Ortega et al. (1998); Arimoto (1996). Many variations of passivity based control are proposed so far Ortega et al. (2002); Zenkov et al. (2003); Duindam and Stramigioli (2004); Golo et al. (2005); Jeltsema and Scherpen (2009); Taniguchi and Fujimoto (2009); Dirksz and Scherpen (2010). The authors proposed the generalized canonical transformation which is a set of coordinate and feedback transformations preserving the port-Hamiltonian structure of the original Fujimoto and Sugie (2001). This technique can be used for trajectory tracking control of port-Hamiltonian systems Fujimoto et al. (2003, 2004).

On the other hand, when we control spacecrafts, it is well known that the typical Euler angle description of the attitude of the vehicle has singular points. In order to prevent them, quaternions are often used Wie (2008); Hughes (1986); Wie and Barba (1985); Kuipers (2002). Since an Euler angle representation is locally described by a three dimensional vector and a quaternion is defined in a four dimensional vector space, the dynamics of the spacecraft is constrained in a subset of the four dimensional quaternion space. This causes difficulty in controlling the system with quaternions. In addition, dynamics with quaternion representation intrinsically becomes nonlinear. Thus the system representation with quaternions have both advantage (no singularity) and disadvantage (difficult to control). Port-Hamiltonian framework and the related techniques allows us to remove some of the difficulties in control.

In the authors former work, we have applied the trajectory tracking control method to port-Hamiltonian systems with quaternions Fujimoto and Nishiyama (2014) in which the main problem is to find an appropriate coordinate transformation to describe the tracking error. This problem was clearly solved but how to ensure asymptotic stability was not clarified as in the conventional simple mechanical systems case. This paper focuses on asymptotic stabilization of port-Hamiltonian systems with quaternions. There already exist some attempts to obtain a general condition for constructing a potential function to stabilize quaternion systems Joshi et al. (2014); Wie (2008) whereas most of them are for system with velocity input. The present paper provides a condition for asymptotic stability using port-Hamiltonian formulation which is more general than the existing results. It is expected to be used not only for stabilization of the attitude but also for more general control objectives for more complex systems.

The organization of the paper is as follows. First of all, we provide a port-Hamiltonian representations for systems with quaternions. Next, the relationship between the error quaternions and the corresponding port-Hamiltonian systems as a preparation for stabilization. Finally, a stabilizing controller with a potential function with stability condition is provided. Furthermore, some numerical examples demonstrate the effectiveness of the proposed method.

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2. PRELIMINARIES

This section briefly refers to the existing results on stabilization of port-Hamiltonian systems and modeling of systems with quaternions.

2.1 Generalized canonical transformations and stabilization

A time-varying port-Hamiltonian system with a Hamiltonian $H(x,t)$ is a system of the form Fujimoto and Sugie (2001)

$$\begin{cases} \dot{x} = (J(x,t) - R(x,t))\dfrac{\partial H(x,t)}{\partial x}^{\mathrm{T}} + g(x,t)u, \\ y = g(x,t)^{\mathrm{T}}\dfrac{\partial H(x,t)}{\partial x}^{\mathrm{T}} \end{cases} \quad (1)$$

with $u, y \in \mathbb{R}^m$, $x \in \mathbb{R}^n$, a symmetric semi-positive definite matrix $R(x,t) = R(x,t)^{\mathrm{T}} \succ 0$, and a skew symmetric matrix $J(x,t) = -J(x,t)^{\mathrm{T}}$. All functions are supposed to be sufficiently smooth.

If the Hamiltonian function $H(x,t)$ has lower bound and $\partial H(x,t)/\partial t \leq 0$ holds then the system (1) is *passive*. Moreover if the system is periodic and *zero-state detectable* then a simple feedback $u = -C(x,t)\,y$ with $C(x,t) \geq \varepsilon I > 0$ renders the system asymptotically stable. The following stabilization procedure is based on this approach.

A generalized canonical transformation Fujimoto and Sugie (2001) is a set of transformations

$$\begin{aligned} \bar{x} &= \Phi(x,t) \\ \bar{H} &= H(x,t) + U(x,t) \\ \bar{y} &= y + \alpha(x,t) \\ \bar{u} &= u + \beta(x,t) \end{aligned} \quad (2)$$

which preserves the structure of port-Hamiltonian systems described in (1), that is, the transformed system has the form

$$\begin{cases} \dot{\bar{x}} = (\bar{J}(\bar{x},t) - \bar{R}(\bar{x},t))\dfrac{\partial \bar{H}(\bar{x},t)}{\partial \bar{x}}^{\mathrm{T}} + \bar{g}(\bar{x},t)\bar{u} \\ \bar{y} = \bar{g}(\bar{x},t)^{\mathrm{T}}\dfrac{\partial \bar{H}(\bar{x},t)}{\partial \bar{x}}^{\mathrm{T}} \end{cases} \quad (3)$$

where $\bar{x}$, $\bar{H}$, $\bar{y}$ and $\bar{u}$ denote the new state, the new Hamiltonian, the new output and the new input, respectively. The properties of such transformations are summarized as follows.

Lemma 1. Fujimoto et al. (2003) (i) Consider the system (1). For any functions $U(x,t) \in \mathbb{R}$ and $\beta(x,t) \in \mathbb{R}^m$, there exists a pair of functions $\Phi(x,t) \in \mathbb{R}^n$ and $\alpha(x,t) \in \mathbb{R}^m$ such that the set (2) yields a generalized canonical transformation. A function Φ yields a generalized canonical transformation if and only if a partial differential equation (PDE)

$$\frac{\partial \Phi}{\partial(x,t)}\begin{pmatrix} (J-R)\dfrac{\partial U}{\partial x}^{\mathrm{T}} + (K-S)\dfrac{\partial(H+U)}{\partial x}^{\mathrm{T}} + g\beta \\ -1 \end{pmatrix} = 0 \quad (4)$$

holds with a skew-symmetric matrix $K(x,t) \in \mathbb{R}^{n\times n}$ and a symmetric matrix $S(x,t) \in \mathbb{R}^{n\times n}$ satisfying $R + S \geq 0$. Further the change of output α and the matrices $\bar{J}$, $\bar{R}$ and $\bar{g}$ are given by

$$\alpha = g^{\mathrm{T}}\frac{\partial U}{\partial x}^{\mathrm{T}} \quad (5)$$

$$\bar{g} = \frac{\partial \Phi}{\partial x}g,\ \bar{J} = \frac{\partial \Phi}{\partial x}(J+K)\frac{\partial \Phi}{\partial x}^{\mathrm{T}},\ \bar{R} = \frac{\partial \Phi}{\partial x}(R+S)\frac{\partial \Phi}{\partial x}^{\mathrm{T}}. \quad (6)$$

(ii) If the system (1) is transformed by the generalized canonical transformation with U, β and S such that $H + U \geq 0$, then the new input-output mapping $\bar{u} \mapsto \bar{y}$ is passive with the storage function $\bar{H}$ if and only if

$$\frac{\partial(H+U)}{\partial(x,t)}\begin{pmatrix} (J-R)\dfrac{\partial U}{\partial x}^{\mathrm{T}} - S\dfrac{\partial(H+U)}{\partial x}^{\mathrm{T}} + g\beta \\ -1 \end{pmatrix} \geq 0. \quad (7)$$

(iii) Suppose moreover that (7) holds, that $H + U$ is positive definite. Then the feedback $u = -\beta - C(x,t)(y + \alpha)$ with $C(x,t) \geq \varepsilon I > 0 \in \mathbb{R}^{m\times m}$ renders the system stable. Suppose moreover that $H+U$ is decrescent, and the transformed system is periodic. Then the feedback renders the system uniformly asymptotically stable.

Using the generalized canonical transformation, we can change the property of the system without changing the intrinsic passive property and stabilize it based on passivity based approach (in Part (iii)). See Fujimoto and Sugie (2001); Fujimoto et al. (2003) for the detail.

Remark 1. In Theorem 1, Part (i) gives a condition for preserving the form of port-Hamiltonian systems, where the functions U and β are free parameters. After those functions are determined, the coordinate transformation $\bar{x} = \Phi(x,t)$ can be obtained by solving the partial differential equation (4). Further, the matrices K and S are free parameter matrices assigning the new structure and dissipation matrices $\bar{J}$ and $\bar{R}$ as in (6). In Part (ii), Condition (7) guarantees the passivity property of the transformed system $\partial \bar{H}(\bar{x},t)/\partial t \leq 0$. Part (iii) is an application of the passivity based stabilizing control $u = -C(x,t)y$, $C(x,t) \geq \varepsilon I > 0$ to the transformed passive system.

2.2 Quaternions

A quaternion is a mathematical object similar to an element of $\mathbb{R}^4$. Since it is related with an Euler angle, it describes the attitude of a rigid body. It has the form of

$$\check{q} = q_1\boldsymbol{i} + q_2\boldsymbol{j} + q_3\boldsymbol{k} + q_4. \quad (8)$$

Here the symbols $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ and 1 are the (linearly independent) basis of quaternions and the corresponding coefficients $q_1, q_2, q_3, q_4 \in \mathbb{R}$ are real numbers. They satisfy

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1. \quad (9)$$

A typical example of a quaternion is a rotation transformation of a rigid body whose normalized rotating axis is $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^{\mathrm{T}} \in \mathbb{R}^3$ with the rotation angle $\theta \in \mathbb{R}$ as follows.

$$\begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} = \begin{pmatrix} \lambda_1 \sin\dfrac{\theta}{2} \\ \lambda_2 \sin\dfrac{\theta}{2} \\ \lambda_3 \sin\dfrac{\theta}{2} \\ \cos\dfrac{\theta}{2} \end{pmatrix}$$