Available online at www.sciencedirect.com **5th IFAC Workshop on Lagrangian and Hamiltonian Methods**

IFAC-PapersOnLine 48-13 (2015) 045–050

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Abstract: A nonlinear control system often has complicated input-to-state relationship, mainly due to its controllability structure in nonlinear sense. A difficult but challenging nature of such systems is that they can only be controlled using appropriate combination of *periodic* inputs, systems is that they can omy be controlled using appropriate combination of *periodic* inputs,
corresponding to the Lie brackets. However, the effect of so-called Lie bracket motion tends to corresponding to the Lie brackets. However, the effect of so-called Lie bracket motion tends to
be limited due to small choice of the input amplitudes. In this paper, focusing on a class of be immed due to small choice of the input amplitudes. In this paper, locusing on a class of
nonholonomic systems as typical examples, we propose to approximate the state displacement under periodic signals with larger amplitudes, and utilize the result to design periodic control under periodic signals with larger amplitudes, and utilize the result to design periodic control
input. The key of our approach is the use of suitable special function, such as the Bessel functions, input. The key of our approach is the use of suitable special function, such as the besself directions,
for series expansion to predict the state displacement, as well as considering symmetry of the for state space. state space. nonholonomic systems as typical examples, we propose to approximate the state displacement for series expansion to predict the state displacement, as well as considering symmetry of the systems is that they can only be controlled using appropriate combination of *periodic* inputs, corresponding to the Lie brackets. However, the effect of so-called Lie bracket motion tends to be limited due to small choice of the input amplitudes. In this paper, focusing on a class of $\frac{1}{2}$ input. The key of our approach is the use of suitable special function, such as the Bessel functions, f_{state} space.

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Keywords: nonlinear control, nonholonomic systems, holonomy, periodic control, averaging technique, mobile robot technique, mobile robot *Keywords:* nonlinear control, nonholonomic systems, holonomy, periodic control, averaging *Keywords:* nonlinear control, nonholonomic systems, holonomy, periodic control, averaging technique, mobile robot technique, mobile robot

1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION

Nonlinear dynamical systems have been providing a lot liarity of nonlinear systems would be their complicated of challenging topics to control engineers. A major pecu-of challenging topics to control engineers. A major pecuof changing to control eigineers. A major pecularity of nonlinear systems would be their complicated marity of nonlinear systems would be then complicated
structure of controllability. In the case of linear timestructure of controllability. In the case of intear time-
invariant systems, the principle of superposition holds, mvariant systems, the principle of superposition notes, the control inputs just go through linear combination of integrator chains, and their controllability is judged by megrator chains, and their controllability is judged by
rank condition of the controllability matrix. In contrast, rank condition of the contronability matrix. In contrast, nonlinear systems often have complicated input-to-state or nonimear systems often have compilicated input-to-state or
input-to-output relationship; it is not easy to describe, in mput-to-output relationship, it is not easy to describe, in
general, where the state variable would reach for the given control inputs. Controllability test for nonlinear systems control inputs. Contronability test for nominear systems are given in various forms of Lie algebra rank condition are given in various forms of the algebra raint condition
(Hermann and Krener (1977); Sussmann (1983); Aeyels (1000) depending on what we expect for controllationly. (1983) depending on what we expect for controllability. (1983) depending on what we expect for controllability. general, where the state variable would reach for the given
general, inputs. Controllability test for paralinger and we Nonlinear dynamical systems have been providing a lot Nonlinear dynamical systems have been providing a lot of challenging topics to control engineers. A major pecu-of challenging topics to control engineers. A major peculiarity of nonlinear systems would be their complicated liarity of nonlinear systems would be their complicated structure of controllability. In the case of linear time-structure of controllability. In the case of linear timeinvariant systems, the principle of superposition holds, invariant systems, the principle of superposition holds, the control inputs just go through linear combination of integrator chains, and their controllability is judged by integrator chains, and their controllability is judged by rank condition of the controllability matrix. In contrast, nonlinear systems often have complicated input-to-state or nonlinear systems often have complicated input-to-state or input-to-output relationship; it is not easy to describe, in input-to-output relationship; it is not easy to describe, in are given in various forms of Lie algebra rank condition (Hermann and Krener (1977); Sussmann (1983); Aeyels (Hermann and Krener (1977); Sussmann (1983); Aeyels (1983) depending on what we expect for controllability. (1983) depending on what we expect for controllability.

Nonholonomic systems form a class of such nonlinear systems, originally defined in classical mechanics as nonsystems, originary defined in classical mechanics as non-
integrable mechanical constraints (Goldstein et al. (2002); megrable mechanical constraints (Goldstein et al. (2002),
Bloch (2003)). Most of nonholonomic systems have multiof them are controllable even without so-called drift term ple control inputs that are coupled to each other, and some ple control inputs that are coupled to each other, and some *f*(*f*) them are controllable even without so-called drift term of them are controllable even without so-called drift term σ (say *f*(*ξ*)), thanks to the Lie bracket between the input $\frac{d}{dx}$ (say $f(\zeta)$), thanks to the Lie bracket between the input
vector-fields (such as $[g_1, g_2](\zeta)$). It is also known that vector-nears (such as $[g_1, g_2]$ (ζ)). It is also known that the system without drift term (driftless systems) are not the system wholut unit term (unitiess systems) are not stabilizable by any smooth time-invariant nonlinear feedstabilizable by any shooth time-invariant nonlinear leed-
back law Brockett (1983) even if they are controllable; this fact motivate us to pursue non-smooth approach (switched fact motivate us to pursue non-smooth approach (switched Nonholonomic systems form a class of such nonlinear integrable mechanical constraints (Goldstein et al. (2002);
 Γ Bloch (2003)). Most of nonholonomic systems have multi-Bloch (2003)). Most of nonholonomic systems have multiple control inputs that are coupled to each other, and some ple control inputs that are coupled to each other, and some of them are controllable even without so-called drift term (say $f(\xi)$), thanks to the Lie bracket between the input (say $f(\xi)$), thanks to the Lie bracket between the input
vector-fields (such as $[g_1, g_2](\xi)$). It is also known that
the system without drift term (driftless systems) are not the system without drift term (driftless systems) are not stabilizable by any smooth time-invariant nonlinear feed-stabilizable by any smooth time-invariant nonlinear feedback law Brockett (1983) even if they are controllable; this back law Brockett (1983) even if they are controllable; this fact motivate us to pursue non-smooth approach (switched fact motivate us to pursue non-smooth approach (switched control, discontinuous control) or partially feedforward approach (time tarying control, periodic control). approach (time-varying control, periodic control). approach (time-varying control, periodic control). control, discontinuous control) or partially feedforward approach (time-varying control, periodic control). approach (time-varying control, periodic control).

A well-known basic principle for periodic control of non-A wen-known basic principle to periodic control of hon-
holonomic systems is so-called *area rule*, which relates the notion systems is so-cannot *after rate*, which relates the Lie brackets with periodic control signals in the sense that proximately parallel to the corresponding Lie bracket and the state change after one cycle of periodic control is ap-the state change after one cycle of periodic control is apis proportional to the area enclosed by the periodic signal proximately parallel to the corresponding Lie bracket and proximately parallel to the corresponding Lie bracket and proximately parallel to the corresponding Lie bracket and is proportional to the area enclosed by the periodic signal (1995)). However, this sort of approxomation is essentially (Murray and Sastry (1993); Leonard and Krishnaprasad (Murray and Sastry (1993); Leonard and Krishnaprasad (1995) . However, this sort of approximation is essentially (1999)). However, this sort or approxomation is essentially
based on Taylor series expansion, thus it does work well based on Taylor series expansion, thus it does work went
for periodic control with small amplitudes, or in the case for periodic control with sinan amplitudes, or in the case
of systems with drift term. From practical points of view, of systems with the drift term. From practical points of view, we often prefer to utilize full range of control amplitude we often prefer to utilize fun range of control ampitude as well as effect of the drift term to save control effort. In as wen as enect of the unit term to save control enort. In
this paper, focusing on a typical example of nonholonomic system with drift term, we simply derive the system sosystem with unit term, we simply derive the system so-
lution under periodic control using series expansion with of large amplitude. Bessel functions, whose approximation is valid for the case Bessel functions, whose approximation is valid for the case or targe amproace. of large amplitude. of large amplitude. (Murray and Sastry (1993); Leonard and Krishnaprasad this paper, focusing on a typical example of nonholonomic
cystem, with drift term, we simply derive the system as A well-known basic principle for periodic control of non-A well-known basic principle for periodic control of nonholonomic systems is so-called *area rule*, which relates the holonomic systems is so-called *area rule*, which relates the
Lie brackets with periodic control signals in the sense that the state change after one cycle of periodic control is ap-the state change after one cycle of periodic control is approximately parallel to the corresponding Lie bracket and proximately parallel to the corresponding Lie bracket and is proportional to the area enclosed by the periodic signal (1000) based on Taylor series expansion, thus it does work well based on Taylor series expansion, thus it does work well for periodic control with small amplitudes, or in the case of systems with drift term. From practical points of view, of systems with drift term. From practical points of view, we often prefer to utilize full range of control amplitude as well as effect of the drift term to save control effort. In as well as effect of the drift term to save control effort. In lution under periodic control using series expansion with Bessel functions, whose approximation is valid for the case Bessel functions, whose approximation is valid for the case of large amplitude. of large amplitude.

This paper is organized as follows. In Section 2, we explain I in paper is organized as follows. In section 2, we explain a motivating problem derived from unnateral folling disk
control. Section 3 describes a limitation of Lie bracketof the target system and a feedforward point-to-point based analysis. In Section 4, we give an explicit solution based analysis. In Section 4, we give an explicit solution based analysis. In section 4, we give an explicit solution
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a motivating problem derived from unilateral rolling disk control. Section 3 describes a limitation of Lie bracketbased analysis. In Section 4, we give an explicit solution based analysis. In Section 4, we give an explicit solution of the target system and a feedforward point-to-point of the target system and a feedforward point-to-point control under given periodic control with several numerical examples. Concluding remarks are given in Section 5. examples. Concluding remarks are given in Section 5.

[⋆] This work was supported by Grant-in-Aid for JSPS Fellows Grant **★ This work was supported by Grant-in-Aid for JSPS Fellows Grant** Number 267770 Number 267770 Number 267770 **★ This work was supported by Grant-in-Aid for JSPS Fellows Grant**

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2. MOTIVATION FROM CYLINDRICAL MOBILE ROBOT

This paper is motivated by the authors' previous work on cylindrical mobile robot (Hirano et al. (2013)). Fig. 1 shows a rolling cylinder put on the horizontal floor, where a point on the rim of the circular section is in contact with the floor (this motion is called *edge rolling*). *R* is the constant radius of the cylinder, (X_C, Y_C) indicates the position of the contact point, ψ is the heading angle and ϕ is the rotation(pitch) angle of the cylinder. The robot itself is supposed to be driven by actuating an internal pendulum. We assume that there is no slip nor slide between the cylinder and the floor, although its spin about the vertical axis passing through the contact point is freely allowed. If we assign the speed of the contact point $R\psi$ as the control input u_1 , and the heading angular velocity ψ as the other control input u_2 , then the system can be viewed as a standard two-wheeled mobile robot (Fig.2). In the edge rolling, the contact point almost always moves along circular loci because of the gyroscopic effect generated by the gravity.

Fig. 1. Cylindrical rolling robot in edge-rolling motion

Fig. 2. Model of the simple two-wheeled robot

Note that, in order to maintain the edge rolling motion, the rotation speed $\dot{\phi}$ and the heading angular velocity should be *certain non-zero values*, say \bar{u}_1, \bar{u}_2 , respectively. This motivates us to consider the following model of nonlinear state equation with drift, where the control inputs $u_1(t)$, $u_2(t)$ can be applied as deviation to the constant bias $\bar{u}_1, \bar{u}_2.$

$$
\dot{\xi} = f(\xi) + g_1(\xi)u_1(t) + g_2(\xi)u_2(t),\tag{1}
$$

$$
\xi(t) = \begin{bmatrix} x \\ y \\ \phi \end{bmatrix}, f(\xi) = \begin{bmatrix} \bar{u}_1 \cos \psi \\ \bar{u}_1 \sin \psi \\ \bar{u}_2 \end{bmatrix},
$$

$$
g_1(\xi) = \begin{bmatrix} \cos \psi \\ \sin \psi \\ 0 \end{bmatrix}, g_2(\xi) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.
$$

3. EFFECT AND LIMITATION OF TAYLOR SERIES APPROACH

In the case of driftless nonlinear state equation

$$
\xi = g_1(\xi)u_1(t) + g_2(\xi)u_2(t),\tag{2}
$$

an elementary approach to control is to apply a pair of periodic control signals as mentioned in the introduction. If we apply the following square periodic wave

$$
(u_1(t), u_2(t)) = \begin{cases} (1,0), & t \in [0,T) \\ (0,1), & t \in [T,2T) \\ (-1,0), & t \in [2T,3T) \\ (0,-1), & t \in [3T,4T] \end{cases}
$$
(3)

to the system(2), then the resulting displacement in the state space is given by

$$
\xi(4T) = \xi(0) + T^2[g_1, g_2](\xi(0)) + O(T^3), \tag{4}
$$

where the Lie bracket is defined as the vector-field

$$
[g_1, g_2](\xi) := \frac{\partial g_2}{\partial \xi} g_1(\xi) - \frac{\partial g_1}{\partial \xi} g_2(\xi). \tag{5}
$$

Eq. (4) implies that, we obtain an extra control direction other than the original input vector-fields $g_1(\xi)$, $g_2(\xi)$ in approximate sense, if $[g_1, g_2](\xi)$ is independent to $g_1(\xi), g_2(\xi)$ at any ξ .

Let us turn to consider the case of systems with drift (1). If we apply the same control (3) to the system (1) , a straightforward computation tells us

$$
\xi(4T) = \xi(0) + 4Tf(\xi(0)) + \frac{1}{2}(4T)^2 \frac{\partial f}{\partial \xi}\Big|_{\xi(0)} f(\xi(0)) + T^2[g_1, g_2](\xi(0)) + 2T^2([g_1, f] + [g_2, f])(\xi(0)) + O(T^3).
$$

The first three terms come from Taylor expansion of $f(\xi)$, while the next three terms are concerned with the Lie bracket. In the case of the system (1),

$$
[g_1, f](\xi) = \begin{bmatrix} \bar{u}_2 \sin \psi \\ -\bar{u}_2 \cos \psi \\ 0 \end{bmatrix}, \quad [g_2, f](\xi) = \begin{bmatrix} -\bar{u}_1 \sin \psi \\ \bar{u}_1 \cos \psi \\ 0 \end{bmatrix},
$$

$$
[g_1, g_2](\xi) = \begin{bmatrix} \sin \psi \\ -\cos \psi \\ 0 \end{bmatrix},
$$

which are all parallel to the direction perpendicular to the orientation of the vehicle. However, what really happens for $\xi(0) = 0$, $u_1(t) = 0.8 \cos t$, $u_2(t) = 0$ is that the vehicle eventually proceeds in the direction parallel to the original orientation, as shown in Fig. 3. This gap makes us aware of fundamental limitation of Taylor-series and Lie-bracket based approach.

4. SOLUTION OF THE TWO-WHEELED VEHICLE WITH DRIFT USING BESSEL FUNCTIONS

4.1 Basic property of Bessel functions

For further analysis, we introduce basic properties of Bessel functions. *Bessel function of the first kind*, origiDownload English Version:

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