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A time domain reconstruction method of randomly sampled frequency sparse signal

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A B S T R A C T

In this paper stochastic sampling as a method of frequency sparse signal acquisition is presented. Basic principle of compressed sensing is reviewed, with emphasis on nonuniform sampling and signal reconstruction methods. A robust time domain reconstruction method of randomly sampled signal through compressed sensing approach is proposed. The presented reconstruction algorithm is evaluated by means of simulations, with comparison to conventional compressed sensing reconstruction and the most common practical issues taken into account. Simulation results indicate that the proposed reconstruction method is resistent to high levels of quantization and uncorrelated noise. Experiments with real hardware were also performed, results of which confirm the ability of stochastic sampling framework to overcome the Nyquist limit of analog-to-digital converters.

1. Introduction

In recent years, compressive sensing (CS) came up as a promising method of sub-Nyquist sampling. It is a powerful tool for signal analysis, and it allows to acquire signals with fewer measurements then previously thought possible. This is accomplished by exploiting the fact that many real-world signals have far fewer degrees of freedom than the signal size might indicate. For instance, spectrally sparse (narrowband) signal depends upon only a few degrees of freedom, although its total bandwidth is exceptionally wide. Nyquist sampling of such signal is difficult, and it requires the storage of significantly large amounts of data. With sparse signals, CS allows for much lower sampling frequencies. The result is lower output data transfer rate, and there is a possibility of overcoming the speed limit of conventional analog-to-digital converters (ADCs). Lower sampling rate by extension also means reduction of power consumption, and lower demands on communication channels. A wide field of possible applications could benefit from these advantages, e. g. wireless sensor networks.

Two most promising methods of CS have been proposed and are discussed in literature, random modulation pre-integration (RMPI), and nonuniform sampling. This work is focused on nonuniform sampling, or more specifically, the stochastic sampling (SS) architecture ([1,2]). This architecture has to be distinguished from adaptive ADCs [3,4] that also perform nonuniform sampling.

RMPI architectures have been extensively studied and successfully used ([5,6]), but they are difficult to implement. Before the signal can be sampled with a lower rate, it has to be conditioned by specialized analog circuitry operating at Nyquist rates [8]. The precise nonideal nature of this circuitry, such as the modulation signal shape [9], jitter [10] and the filter impulse response [11] need to be precisely modelled in order to ensure low reconstruction error. SS is very promising in this regard, because there is no circuitry required besides the ADC. Peak sampling frequency of this ADC may be lower than the Nyquist rate, and the signal can be reconstructed assuming its frequency band is known. Certain applications could also benefit from the fact that the SS framework is inherently resistant to data loss [12].

The implementation of random sampler and the reconstruction algorithm will be discussed, from which the latter is not extensively covered by literature. Evaluation of the proposed algorithm is presented, with regards to practical issues such as inherent and quantization noise.

2. The principle of compressed sensing

The main goal of CS is transforming a signal to a suitable descriptive domain and condensing it into a few samples, which is referred to as analog-to-information conversion (AIC). This compressed information is then sent to the receiver, where the original signal is reconstructed. No information loss and exact reconstruction is theoretically possible with CS, with high potential compression ratios.

In order for CS to be applicable, it is assumed that the input signal can be represented by a linear combination of known basis functions.

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Furthermore, the signal must be *sparse*, meaning that it consists of only a small number of basis functions. The signal composed of s basis functions is denoted to be s-sparse. Let us define L basis vectors $\boldsymbol{\psi}_l \in \mathbb{R}^{N\times 1}$, $1 \leqslant l \leqslant L$ to represent any possible input signal components. The columns of the basis matrix $\boldsymbol{\Psi} \in \mathbb{R}^{N\times L}$ consist of these basis vectors. The input signal vector $\mathbf{f} \in \mathbb{R}^{N\times 1}$ can then be described as

$$\mathbf{f} = \boldsymbol{\Psi}\mathbf{x} \tag{1}$$

where the s-sparse vector $\mathbf{x} \in \mathbb{R}^{L\times 1}$ conducts the linear combination [13].

2.1. Analog-to-information conversion

Conventional ADCs acquire signal samples at equidistant time instants, according to the Nyquist sampling theorem. With signals highly sparse at a certain domain, such sampling means great redundancy in acquired data, since the amount of information contained within a sparse signal is limited. AIC exploits this fact, and only takes a limited number of samples, sufficient to represent the information content. High bandwidth sparse signals can be precisely measured by time-equivalent sampling as well, and reconstructed by a very simple algorithm [14]. This approach however relies on the signal being stationary for a prolonged time period. CS employs different sampling and reconstruction strategies and in general does not require the measured signal to be stationary.

The input signal $\mathbf{f} \in \mathbb{R}^{N\times 1}$ is correlated with $M < N$ measurement signals, represented by M rows of the measurement matrix $\boldsymbol{\Phi} \in \mathbb{R}^{M\times N}$. This performs the AIC, with resulting information signal vector

$$\mathbf{y} = \boldsymbol{\Phi}\mathbf{f} \in \mathbb{R}^{M\times 1} \tag{2}$$

Let the input signal vector be s-sparse on basis $\boldsymbol{\Psi}$, and let bases $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ be incoherent (rows $\boldsymbol{\varphi}_m$ of $\boldsymbol{\Phi}$ do not sparsely represent the columns $\boldsymbol{\psi}_l$ of $\boldsymbol{\Psi}$, and vice versa). If $s < M \ll N$, input signal can be reconstructed [15]. According to [13], an approximation of minimal M needed is given by

$$M_{\min} = \mu s \log_{10}(N) \tag{3}$$

assuming zero or negligible noise [16]. $\mu = 1$ for incoherent bases, but has to be increased to $\mu = L/N$ if $N < L$. By inserting (1) into (2) the information signal becomes

$$\mathbf{y} = \boldsymbol{\Phi}\boldsymbol{\Psi}\mathbf{x} \tag{4}$$

with the reconstruction matrix

$$\mathbf{A} = \boldsymbol{\Phi}\boldsymbol{\Psi} \in \mathbb{R}^{M\times L}. \tag{5}$$

2.2. Reconstruction

At the receiving end of CS framework, the information signal $\mathbf{y}$, and both bases $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ are known. The only unknown required for reconstruction of (1) is $\mathbf{x}$. If $\mathbf{A}$ was a square matrix (which would mean applying conventional Nyquist sampling), the problem could be solved simply by inverting $\mathbf{A}$. But since $\mathbf{A}$ is a rectangular matrix, it cannot be simply inverted, and an undetermined system of M equations and L unknowns is to be solved. Here the importance of sparsity turns out, because based on this requirement a unique solution can be found. Out of all the possible solutions, the right solution is the one that is the most sparse [17].

The sparsity s of vector $\mathbf{x}$ is in general its ℓ_0 pseudo-norm, defined as the number of non-zero elements:

$$\ell_0 = \|\mathbf{x}\|_0 := \{i\colon x_i \neq 0\} \tag{6}$$

Sparsity is an additional information, used to construct the measurement matrix $\boldsymbol{\Phi}$, and forming an optimization problem

$$\min\|\mathbf{x}\|_0 \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{y} \tag{7}$$

Based on the ideal definition of sparsity (6), it should be trivial to find s simply by performing the AIC and counting the number of non-zero elements of $\mathbf{y}$. However, for any real signal with inherent noise, there are a few elements of large value, and other elements small but not zero [18]. To address this issue, other metrics were proposed [19]. In [20] it was deduced, that with highly sparse signals the norm

$$\ell_1\colon \|\mathbf{x}\|_1 = \sum_{i=1}^{n} |x_i| \tag{8}$$

can be used, which is preferable since it leads to a convex optimization problem

$$\min\|\mathbf{x}\|_1 \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{y} \tag{9}$$

This guarantees a correct optimal solution, if such exists. Moreover, efficient algorithms may be used – with the highly unstable ℓ_0 norm, only brute force methods are successful. With complying to (9) the original signal estimate can be found as

$$\hat{\mathbf{f}} = \boldsymbol{\Psi}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{y} \tag{10}$$

where $(*)^{-1}$ denotes the pseudoinverse matrix of $*$.

Another issue with CS is determining the base matrices $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$. For some signals, their sparse domain is known, e. g. frequency or wavelet. For other there is not a specific domain for this purpose, and a suitable dictionary must be learned. Correctly constructed set $\boldsymbol{\Psi}$ of basic functions ensures high compression ratio and low reconstruction error.

As for the measurement matrix $\boldsymbol{\Phi}$, pseudo-random elements are used to ensure the incoherence of bases. With CS architectures such as RMPI ([5–7,9–11]), measurement matrix consists of randomly placed $\pm$ 1's and it must be full-rank. The *SS* architecture simply takes samples of the input signal at random time instants. For the implementation of SS, only a vector containing random sequence of 0's and 1's is needed. To be consistent with mathematical descriptions above, measurement matrix of SS would be a full-rank matrix of 0's, with a 1 placed in each row. Positions of 1's are random, but their column indices must be ascending. CS framework with both sensing and decoding side is summarized in Fig. 1.

3. Stochastic sampling

In order to subsample the Nyquist grid, SS exploits the incoherence of time and frequency. Subsampling is random – time instants at which the signal is sampled are selected randomly. Thus there is no coherent aliasing effect, meaning no frequency information loss, and the original signal can be recovered via nonlinear processing [1,2]. A conceptual diagram of SS is shown in Fig. 2.

SS takes Nyquist-rate samples of input signal and randomly discards most of the samples according to pseudo-random bit sequence (PRBS). Sampling is therefore random, but samples are spaced with integer multiples of underlying Nyquist rate. Additional digital signal processing (DSP) is applied in order to encode both sample values and their positions into the output data stream. Reduced power consumption and higher input bandwidth of SS may be achieved by using ADC triggered by PRBS, rather than discarding samples of a conventional ADC.

Let $F[\omega]$, $\omega = 2\pi\nu$ be the discrete Fourier transform (DFT) of

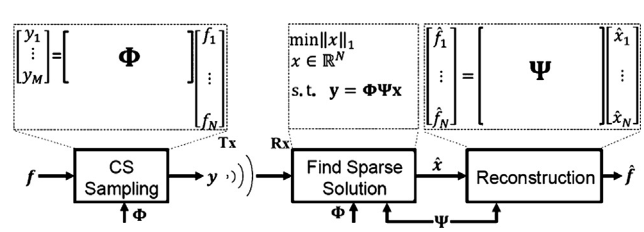

Fig. 1. Compressed sensing framework.