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Energy shaping for the robust stabilization of a wheeled inverted pendulum

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Abstract: The paper deals with the robust energy-based stabilization of a wheeled inverted pendulum, which is an underactuated, unstable mechanical system subject to nonholonomic constraints. The equilibrium to be stabilized is characterized by the length of the driven path, the orientation, and the pitch angle. We use the method of Controlled Lagrangians which is applied in a systematic way, and is very intuitive, for it is physically motivated. After a detailed presentation of the model under nonholonomic constraints, we provide an elegant solution of the matching equations for kinetic and potential energy shaping for the considered systems. Simulations show the applicability and robustness of the method.

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1. INTRODUCTION

The *wheeled inverted pendulum* (WIP) – and its commercial version, the Segway [2015, Jan] – has gained interest for human assistance and transportation in the past several years due to its high maneuverability and simple construction (see, e. g., Li et al. [2013]). A WIP – shown from the side in Figure 1 (left) – consists of a vertical body with two coaxial driven wheels mounted on the body. The actuation of both wheels in the same direction generates a forward (or backward) motion; opposite wheel velocities lead to a turning motion around the vertical axis. Mobile robotic systems based on the WIP like the intelligent two wheeled road vehicle *B2* presented by Baloh and Parent [2003], or novel and more car-like systems like the *Segway* Puma [2015, Jan] are being developed to be used as new personal urban transportation systems. Some institutes have also developed their own WIPs for research purposes, e. g., *Yamabico Kurara*, introduced by Ha and Yuta [1996], or *JOE*, presented by Grasser et al. [2002], to give only some examples. These systems can be further used as service robots like *KOBOKER* (see Lee and Jung [2011]).

The stabilization and tracking control for the WIP is challenging: The system belongs to the class of underactuated mechanical systems, since the number of control inputs is less than the number of degrees of freedom. Furthermore, the upward position of the body represents an unstable equilibrium which needs to be stabilized by feedback. In addition, the system motion is restricted by nonholonomic (nonintegrable) constraints (Bloch [2003]). These constraints do not restrict the configuration space $\tilde{\mathcal{Q}}$ on which the dynamics evolve, but the motion direction at a given point: Because of the rolling-without-slipping constraint it is not possible to move sideways, and the forward velocity of the WIP and its yaw rate are directly given by the angular velocity of the wheels. The goal of this paper is to present the design of a robust nonlinear position controller using energy shaping techniques for wheeled inverted pendulum systems.

1.1 Existing work

Several control laws have been applied to the WIP, mostly using linearized models (see Li et al. [2013], Ha and Yuta [1996], Grasser et al. [2002]). During the last decade, however, researchers have put a strong focus on the nonlinear model for control purposes: Some accessibility and controllability analysis of the WIP has been done by Pathak et al. [2005] and Nasrallah et al. [2007]. Based on the analysis of the nonlinear system, nonlinear control strategies have been developed for Segway-like systems. Pathak et al. [2005] present, e. g., two different two-level controllers based on the partially feedback linearized model for position and velocity control while maintaining stable pitch dynamics; Nasrallah et al. [2007] design in several steps a posture and velocity control for the WIP moving on an inclined plane. Many other types of modeling and control approaches have also been implemented and tested: For a very complete overview of the existing work on modeling and control of WIPs until 2012 the reader is referred to Chan et al. [2013].

Energy shaping techniques, like the method of Controlled Lagrangians, or Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC), have been successfully used for the stabilization of underactuated mechanical systems in the past, see, e. g., Ortega et al. [2002], Chang et al. [2002]. These methods are attractive since they shape the energy of the system but preserve its physical structure, and thus, appear *natural*. The idea of shaping the energy can also be expanded to mechanical systems subject to nonholonomic constraints: Maschke and Van der Schaft [1994] stabilize nonholonomic systems by shaping the potential energy. Muralidharan et al. [2009] stabilize the pitch dynamics of the WIP through IDA-PBC.

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Nonholonomic systems violate one of the necessary conditions for asymptotic stabilization by smooth state feedback formulated by Brockett [1983]. Thus, for the asymptotic stabilization of a desired configuration $q \in \tilde{\mathcal{Q}}$, a discontinuous or time-varying control law is required (Astolfi [1996]). In this paper, to avoid this issue, instead of working in the WIPs six-dimensional configuration space $\tilde{\mathcal{Q}}$, we restrict our analysis to the three dimensional space $\mathcal{Q}$ with local coordinates consisting of the path length, the pitch, and the yawing angle: $\xi = [s\ \alpha\ \theta]^T \in \mathcal{Q}$. The pitch angle is physically restricted to $-\pi/2 < \alpha < \pi/2$. We design a passivity-based controller for the stabilization of an equilibrium $\xi^* \in \mathcal{Q}$. The controller is thereafter parametrized applying local linear dynamics assignment (LLDA), a method used to fix design parameters in nonlinear passivity based control by making use of the linearized model (Kotyczka [2013]). Using this approach, prescribed local dynamics (in terms of the closed-loop eigenvalues) can be achieved.

The passivity-based controller presented in this note can be systematically computed and leads to an asymptotically stable equilibrium $\xi^* \in \mathcal{Q}$ with a large domain of attraction. Since the closed-loop mechanical energy is used as Lyapunov function, the framework is remarkably intuitive for it is physically motivated. Moreover, LLDA allows for transparency concerning parameter tuning. The applicability, performance, and robustness of the developed controller is shown with a series of simulations.

Notation: For compactness of notation, the operator $\nabla_x f(x)$ is used to denote the transposed Jacobian of a vector-valued function $f(x)$. Additionally, we will use the notation $\mathrm{s}(\alpha) = \sin\alpha$, and $\mathrm{c}(\alpha) = \cos\alpha$. When obvious from the context, arguments are omitted for simplicity.

2. MODELING

In a mechanical system with nonholonomic constraints, the n-dimensional manifold $\tilde{\mathcal{Q}}$ is the configuration space, its tangent bundle $T\tilde{\mathcal{Q}}$ is the velocity phase space and a smooth (nonintegrable) distribution $\mathcal{D} \subset T\tilde{\mathcal{Q}}$ represents the constraints. The Lagrangian L is a map $L : T\tilde{\mathcal{Q}} \to \mathbb{R}$ and is defined as the kinetic energy minus the potential energy $L = T - V$. A curve $q(t)$ is said to satisfy the constraints if $\dot{q}(t) \in \mathcal{D}_q$, for all $q \in \tilde{\mathcal{Q}}$ and all times t. For k nonholonomic constraints, the admissible velocities in a point q are thus restricted to a $(n-k)$-dimensional subset ($\mathcal{D}_q \cong \mathbb{R}^{n-k}$) of the tangent space $T_q\tilde{\mathcal{Q}}$. The constraint distribution $\mathcal{D}$ is assumed to be regular, i. e., of constant rank. The widely used Lagrange-d'Alembert equations (see, e. g., Bloch [2003])

$$\frac{d}{dt}(\nabla_{\dot{q}} L) - \nabla_q L = A(q)\lambda + \sum F_{ext} \tag{1}$$

describe the dynamics of systems subject to k nonholonomic (Pfaffian) constraints of the form

$$A^T(q)\dot{q} = 0. \tag{2}$$

Assuming there are no external forces other than the input torques $\tilde{\tau}$, (1) results in

$$\tilde{M}(q)\ddot{q} + \tilde{C}(q,\dot{q})\dot{q} + \nabla_q V(q) = \tilde{\tau} + A(q)\lambda, \tag{3}$$

where $\tilde{M} = \tilde{M}^T$ is the positive definite mass matrix, and the term $\tilde{C}\dot{q}$ represents the Coriolis and centripetal forces. The constraints have been adjoined to the system using Lagrange multipliers $\lambda \in \mathbb{R}^k$ that represent the magnitude of the constraint forces which oblige the system to satisfy the constraints. The work done by these forces vanishes as can be seen by looking at the corresponding power

$$P_{constr} = \dot{q}^T A\lambda = \lambda^T A^T \dot{q} = 0. \tag{4}$$

The approach, as explained in the following, is also used, e. g., by Pathak et al. [2005] for the modeling of the WIP: Due to the nonholonomic constraints (2), the admissible velocities at $q \in \tilde{\mathcal{Q}}$ must be of the form

$$\dot{q} = S(q)\nu, \tag{5}$$

with a smooth full rank matrix S satisfying $A^T S = 0$ for all $q \in \tilde{\mathcal{Q}}$, and local coordinates of the constrained tangent space $\nu \in \mathcal{D}_q$. The admissible velocities at q lie in the subspace of $T_q\tilde{\mathcal{Q}}$ spanned by the columns of S, which is nothing but the $(n-k)$-dimensional space $\mathcal{D}_q$. Now, replace $\dot{q} = S\nu$ and $\ddot{q} = S\dot{\nu} + \dot{S}\nu$ in (3), and eliminate the constraints by pre-multiplying it by S^T

$$S^T\tilde{M}S\dot{\nu} + S^T\left(\tilde{M}\dot{S} + \tilde{C}S\right)\nu + S^T\nabla_q V = S^T\tilde{\tau}. \tag{6}$$

The dynamical system represented by (6) can also be written in the form

$$\hat{M}\dot{\nu} + \hat{C}\nu + S^T\nabla_q V = \hat{\tau} + \hat{J}\nu, \tag{7}$$

where $\hat{M} = S^T\tilde{M}S$, and $\hat{\tau} = S^T\tilde{\tau}$. Since the matrix $\hat{C}$ is solely defined by the *Christoffel symbols* of $\hat{M}$, the matching of the systems (6) and (7) requires, in general, additional gyroscopic forces $\hat{J}\nu$, where $\hat{J} = -\hat{J}^T$, which are mistakently missing in Muralidharan et al. [2009] for imposing the constraints before taking variations in the derivation of the equations of motion (see Bloch [2003]).

2.1 The wheeled inverted pendulum (WIP)

Different modeling approaches for WIPs can be found, e. g., in Pathak et al. [2005], Delgado et al. [2015], Nasrallah et al. [2007]. The dynamic parameters needed for the modeling of the WIP are listed below in Table 1 with the values used for the simulations. Figure 1 shows

m_B	body mass	1 kg
m_W	wheel mass	0.5 kg
r	wheel radius	0.05 m
b	distance from the wheel axis to the body's center of mass	0.08 m
d	half of the wheel distance	0.05 m
I_B	body's moment of inertia	
$I_{B_{xx}}$	around x-axis	1E-5 kg m^2
$I_{B_{yy}}$	around y-axis	9E-4 kg m^2
$I_{B_{zz}}$	around z-axis	4E-4 kg m^2
I_W	wheel's moment of inertia	
$I_{W_{yy}}$	around y-axis	1E-8 kg m^2
$I_{W_{zz}}$	around z-axis	1E-6 kg m^2
g	gravity constant	10 m/s^2

Table 1. System parameters

a simple scheme of the wheeled inverted pendulum. Let $\tilde{\mathcal{Q}} = \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$ be the configuration space and define local coordinates $q = (x, y, \theta, \alpha, \varphi_l, \varphi_r) \in \tilde{\mathcal{Q}}$. The coordinates φ_l and φ_r represent the absolute rotation of the left and right wheel, respectively. The equations

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