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Control by Interconnection of Distributed Port-Hamiltonian Systems Beyond the Dissipation Obstacle

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Abstract: The main contribution of this paper is a general methodology for the definition of a new passive output that is instrumental for the stabilisation of a large class of distributed port-Hamiltonian systems defined on a one-dimensional spatial domain. This new output is in fact employed within the control by interconnection via Casimir generation paradigm for the synthesis of boundary stabilising control laws. It is well-known that this control technique is limited by the so-called "dissipation obstacle" when the passive controller is interconnected to the natural input/output port of the plant. When it is the case, it is impossible to shape the energy of the system along the directions in which dissipation is present. In this paper, it is shown how these limitations can be removed by interconnecting the boundary controller to the new passive output of the system, and then how the control by interconnection can easily deal with the dissipation obstacle. The general theory is illustrated with the help of a concluding example, the boundary stabilization of the shallow water equation.

Keywords: distributed port-Hamiltonian systems, control by interconnection, boundary control, passivity-based control

1. INTRODUCTION

Port-Hamiltonian systems have been introduced about twenty years ago to describe lumped parameter physical systems in an unified manner. Further information can be found e.g. in van der Schaft [2000], Duindam et al. [2009], van der Schaft and Jeltsema [2014], and also in Macchelli [2014] as far as an extension to macro-economic systems is concerned. The generalisation to the infinite dimensional scenario leads to the definition of distributed port-Hamiltonian systems (see e.g., van der Schaft and Maschke [2002], Macchelli and Maschke [2009]), that have proved to represent a powerful framework for modelling, simulation and control physical systems described by PDEs. Distributed port-Hamiltonian systems share analogous geometric properties with their finite dimensional counterpart, and also the development of stabilising control laws follows the same rationale. Since in most of the cases the Hamiltonian defines the total energy of the system, stabilisation could be obtained by driving the latter to a desired point.

Most of the current research on the stability and stabilisation of distributed port-Hamiltonian systems deals with the development of boundary controllers. For example, in Rodriguez et al. [2001], Macchelli and Melchiorri [2004, 2005], Pasumarthy and van der Schaft [2007], Schöberl and Siuka [2013], this task has been accomplished by generating, a set of Casimir functions in closed-loop that independently from the Hamiltonian functions relates the state of the infinite dimensional port-Hamiltonian system with the state of the controller, which is a finite dimensional port-Hamiltonian system interconnected to the boundary of the distributed parameter one. The shape of the closed-loop energy function is changed by acting on the Hamiltonian of the controller e.g. to introduce a minimum in a desired configuration. As discussed in van der Schaft [2000], Ortega et al. [2001], this procedure is the generalisation of the control by interconnection via Casimir generation (energy-Casimir method) developed for finite dimensional systems. The result is an energy-balancing passivity-based controller that is not able to deal with equilibria that require an infinite amount of supplied energy in steady state, i.e. with the so-called "dissipation obstacle."

In finite dimensions, the dissipation obstacle has been solved within the control by interconnection and Casimir generation paradigm by defining a *new* input/output port to which the passive regulator has to be interconnected. More precisely, a new family of passive outputs has been defined for the original system in such a way that, in closed-loop, a new set of Casimir functions that can be employed in the energy-shaping procedure is present. More details e.g. in Jeltsema et al. [2004], Venkatraman and van der Schaft [2010], Ortega and Borja [2014]. In this paper, such approach has been extended to the distributed parameter scenario. In particular, a new family of passive outputs has been defined for a large class of distributed

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port-Hamiltonian systems with internal dissipation. These new passive outputs are instrumental for removing the dissipation obstacle, i.e. to obtain a closed-loop system characterised by a certain number of Casimir functions that can be used to properly shape the energy of the system and achieve the desired stability properties, i.e. to have a (possibly global) minimum at the desired equilibrium configuration. The general theory is illustrated with the help of a concluding example, i.e. the boundary stabilization of the shallow water equation in which (linear) dissipation is present to model internal friction forces.

2. A CLASS OF DISTRIBUTED PORT-HAMILTONIAN SYSTEMS

In this paper, we refer to the class of distributed port-Hamiltonian systems described by the following PDE:

$$\frac{\partial x}{\partial t}(t,z) = P_1 \frac{\partial}{\partial z}\frac{\delta H}{\delta x}(x(t,z)) + (P_0 - G_0)\frac{\delta H}{\delta x}(x(t,z)) \quad (1)$$

Such class generalizes what has been studied in Le Gorrec et al. [2005], Jacob and Zwart [2012] as far as the linear case is concerned. Here, the spatial domain is $Z = [a, b]$ and $x \in L_2(a, b; \mathbb{R}^n)$ denotes the state (energy) variable. Moreover, $P_1 = P_1^{\mathrm{T}} > 0$, $P_0 = -P_0^{\mathrm{T}}$, and $G_0 = G_0^{\mathrm{T}} \geq 0$, while H is the Hamiltonian (e.g., the total energy) of the system, that is not necessarily quadratic in the energy variables. Finally, δ denotes the variational derivative; refer to van der Schaft and Maschke [2002] for further details.

To define a distributed port-Hamiltonian system, the PDE (1) has to be "completed" with a boundary port. In this respect, the boundary port variables associated to (1) are the vectors $f_\partial, e_\partial \in \mathbb{R}^n$ defined by

$$\begin{pmatrix} f_\partial \\ e_\partial \end{pmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix} P_1 & -P_1 \\ I & I \end{pmatrix}}_{=:R} \begin{pmatrix} \frac{\delta H}{\delta x}(b) \\ \frac{\delta H}{\delta x}(a) \end{pmatrix}$$

which turn out to be a linear combination of the restrictions on the boundary of the spatial domain of the co-energy variables. To have a so-called boundary control system e.g. in the sense of Curtain and Zwart [1995], inputs and outputs have to be defined. From Le Gorrec et al. [2005], a simple procedure to have system (1) in impedance form is the following. Let W and $\tilde{W}$ a pair of $n \times 2n$ full rank real matrices, such that $(W^{\mathrm{T}}\ \tilde{W}^{\mathrm{T}})$ is invertible, and

$$W\Sigma W^{\mathrm{T}} = 0 \qquad W\Sigma\tilde{W}^{\mathrm{T}} = I \qquad \tilde{W}\Sigma\tilde{W}^{\mathrm{T}} = 0 \quad (2)$$

being $\Sigma = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$. The (boundary) input u and output y can be defined as

$$u(t) = W\begin{pmatrix} f_\partial(t) \\ e_\partial(t) \end{pmatrix} \qquad y(t) = \tilde{W}\begin{pmatrix} f_\partial(t) \\ e_\partial(t) \end{pmatrix} \quad (3)$$

and it is easy to prove that the following energy balance equation is satisfied:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\int_a^b \frac{\delta^{\mathrm{T}} H}{\delta x} G_0 \frac{\delta H}{\delta x}\,\mathrm{d}z + y^{\mathrm{T}}u \leq y^{\mathrm{T}}u \quad (4)$$

in which the integral term takes into account the dissipative effects along the spatial domain, and $y^{\mathrm{T}}u$ the supplied power at the boundary port.

3. CONTROL BY INTERCONNECTION AND ENERGY SHAPING

Let us consider a finite dimensional control system in a port-Hamiltonian form:

$$\begin{cases} \dot{x}_c(t) = J_c \frac{\partial H_c}{\partial x_c}(x_c(t)) + G_c u_c(t) \\ y_c(t) = G_c^{\mathrm{T}} \frac{\partial H_c}{\partial x_c}(x_c(t)) \end{cases} \quad (5)$$

Here, $x_c \in \mathbb{R}^{n_c}$ is the state variable, while $J_c = -J_c^{\mathrm{T}}$, G_c and H_c are the to-be-assigned interconnection and input matrices, and Hamiltonian of the controller. The regulator is interconnected in power conserving way to the boundary port (y, u) of (1), i.e.:

$$\begin{pmatrix} u \\ y \end{pmatrix} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}\begin{pmatrix} u_c \\ y_c \end{pmatrix} + \begin{pmatrix} u' \\ 0 \end{pmatrix} \quad (6)$$

with u' a further control input, to obtain a closed-loop system with a Hamiltonian $H_d(x, x_c) = H(x) + H_c(x_c)$. In this way, it is possible to shape the closed-loop energy function to introduce a minimum in the desired equilibrium configuration by acting on the controller Hamiltonian. As in finite dimensions (see van der Schaft [2000], Ortega et al. [2001]), this procedure is facilitated once a proper set of invariants, namely Casimir functions, has been introduced by selecting the controller structure in an appropriated manner. Such invariants allows to relate the state variable of the controller to the state variable of the plant independently from the Hamiltonian functions.

Definition 1. Consider the (autonomous) port-Hamiltonian system resulting from the power-conserving interconnection (6) of (1) and (5), and assume $u' = 0$. A Casimir function is a function $C : X \times \mathbb{R}^{n_c} \to \mathbb{R}$, such that $\dot{C}(x(t,\cdot), x_c(t)) = 0$ along the solutions for every possible choice of H and H_c.

Here, we will look for Casimir functions in the form:

$$C(x(t,\cdot), x_c(t)) = \Gamma^{\mathrm{T}} x_c(t) + \Psi(x(t,\cdot)) \quad (7)$$

with $\Gamma \in \mathbb{R}^{n_c}$, and $\Psi : X \to \mathbb{R}$ a functional of x. Since this function is invariant, for every possible choice of the controller Hamiltonian H_c, a structural algebraic relation between state of the plant and of the controller is present and can be exploited to properly shape the closed-loop Hamiltonian. The characterizations of the possible Casimir functions (7) in closed-loop is given in the following proposition (see e.g. Macchelli [2013], Macchelli et al. [2014]).

Proposition 2. Consider the distributed port-Hamiltonian system with dynamics expressed by the PDE (1), input and output u and y defined in (3), the controller (5), and the power-conserving interconnection (6) with $u' = 0$. Then, (7) is a Casimir function if and only if:

$$P_1 \frac{\partial}{\partial z}\frac{\delta \Psi}{\delta x}(x) + (P_0 + G_0)\frac{\delta \Psi}{\delta x}(x) = 0 \quad (8)$$

$$J_c \Gamma + G_c \tilde{W} R \begin{pmatrix} \frac{\delta \Psi}{\delta x}(b) \\ \frac{\delta \Psi}{\delta x}(a) \end{pmatrix} = 0 \quad (9)$$

$$G_c^{\mathrm{T}} \Gamma + W R \begin{pmatrix} \frac{\delta \Psi}{\delta x}(b) \\ \frac{\delta \Psi}{\delta x}(a) \end{pmatrix} = 0 \quad (10)$$