Measurement 128 (2018) 388–392

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Closely spaced modes identification through modified frequency domain decomposition

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ARTICLE INFO

Keywords:
Frequency domain decomposition
Closely spaced mode
Power spectral density
Singular value decomposition
Orthogonality

ABSTRACT

In practical engineering, power spectral density is always used to identify structural dynamic properties by picking peak points. However, the peaks in the spectral curve are not easy to be determined sometimes. A lot of peaks in the curve would cause some peaks close, which induces the closely spaced modes. The damping ratio is another reason to generate closely spaced modes, because the damping can make some peaks merge together. This paper proposes an innovative method to identify the closely spaced modes. First, the formulation is derived to reveal the components of the closely spaced modes in the power spectral density based on frequency domain decomposition. Then singular vector comparison is presented to determine whether there are closely spaced modes or not, where an angle criterion between two vectors is proposed. Finally, a numerical example is used to validate the effectiveness of the proposed method. The results show that the angle criterion can find the independent and dependent modes. For the close peak points in the spectral curve, the angle can determine that the points are single or closely spaced modes. Therefore, the proposed method to identify the closely spaced modes is efficient.

1. Introduction

Structural modal parameters are identified in some practical engineering, which can reflect the structural properties. The modal parameters can be used to estimate structural response, detect structural damage [1,2], update structural model [3] and optimize sensor deployments. Li et al. [4] proposed the innovative work to estimate the responses of the nonlinear structure by nonlinear modal superposition. Yi et al. [5] presented an intelligent algorithm called the distributed wolf algorithm to optimize the sensor position. To identify the structural modal parameters, there are a lot of methods that contain time domain methods [6–8] and frequency domain methods [9,10]. The time domain method is mainly based on state space dynamic model to derive the identification procedures. The frequency domain is mainly referred to the idea of Fourier transform to obtain the modes without the formation in time domain. Qu et al. [11] proposed the moving data method and a new criterion based on eigensystem realization algorithm to distinguish spurious modes in time domain. Qu et al. [12] presented a modal parameter identification method for practical bridges based on higher-order spectrum in frequency domain. Recently, blind source separation technique is developed in time or frequency domain, which can identify modal shapes first [13].

For some particular structures, the symmetrical architecture or larger damping would generate the closely spaced modes, which has the phenomenon that the modes are close and hard to be distinguished. This problem has attracted many research efforts. Tan et al. [14] developed an integrated method based on continuous wavelet transform and pattern search to identify the closely spaced modes. Kim and Chen [15] introduced a new method to extract the intrinsic mode functions through the filtering algorithm according to the wavelet packet decomposition. Le and Caracoglia [16] estimated high-order and closely-spaced modes through wavelet transform method. Hwang and Kim [17] proposed an optimization method according to the power spectrum to decompose the closely distributed modes for the structure with non-classical damping. Kordkheili et al. [18] experimentally identified the closed spaced modes using both natural excitation technique and eigensystem realization algorithm. The methods are mainly based on time domain or time-frequency domain. In practical engineering, the mode identification in frequency domain has attracted many researchers and engineers. Because the concept that looks like the well-known Fourier transform theory is very easy to be accepted for most of people.

Based on frequency domain decomposition, this paper proposes an innovative method to identify the closely spaced modes, which calculates the angles between singular vectors at the peak point and the ones

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https://doi.org/10.1016/j.measurement.2018.07.006
Received 1 May 2018; Received in revised form 27 June 2018; Accepted 3 July 2018
Available online 05 July 2018

around the peak point. It is different from the previous studies which are based on the wavelet transform method or intrinsic mode functions. The proposed method has more explicit physical meaning. This paper is organized as follows. Section 2 introduces the theory of frequency domain decomposition method, and simplifies the formulation for the individual mode. Section 3 derives the formulation of the power spectral density for closely spaced modes, and reveals the influence on power spectral density. Finally, the proposed method is proved by a numerical example.

2. Theoretical background of frequency domain decomposition

Considering an n degree-of-freedom (DOF) structure, the structural response is $y(t)$ subjected to the excitation $f(t)$. Then the frequency response function (FRF) of the structure can be estimated as Eq. (1) according to Parseval's theorem.

$$\mathbf{G}_{yy}(j\omega) = \mathbf{H}(j\omega)\mathbf{G}_{xx}(j\omega)\mathbf{H}^{\mathrm{H}}(j\omega) \tag{1}$$

where $\mathbf{G}_{yy}(j\omega)$ is the power spectral density matrix of $y(t)$; $\mathbf{G}_{xx}(j\omega)$ is the power spectral matrix density of $f(t)$; the superscript "H" means conjugate transpose; $\mathbf{H}(j\omega)$ is the FRF that can be also represented as Eq. (2).

$$\mathbf{H}(j\omega) = \sum_{r=1}^{n}\left[\frac{\mathbf{R}_r}{j\omega-\lambda_r} + \frac{\mathbf{R}_r^*}{j\omega-\lambda_r^*}\right] \tag{2}$$

with

$$\lambda_r = -\zeta_r\omega_r + j\omega_{dr} = -\zeta_r\omega_r + j\omega_r(1-\zeta_r^2)^{1/2} \tag{3}$$

$$\mathbf{R}_r = \boldsymbol{\varphi}_r\boldsymbol{\Gamma}_r^{\mathrm{T}} \tag{4}$$

$$\boldsymbol{\varphi}_r = [\varphi_{1r}\varphi_{2r}\cdots\varphi_{nr}]^T \tag{5}$$

$$\boldsymbol{\Gamma}_r = [\Gamma_{1r}\Gamma_{2r}\cdots\Gamma_{nr}]^T \tag{6}$$

where the DOF number n is also the mode order; the superscript "T" means transpose; the superscript "$*$" means conjugate; $\lambda_r$is the r-th order pole that is expressed as Eq. (3); ζ_r is the damping ratio; ω_r and ω_{dr} are the natural frequencies without and with consideration of damping, respectively; $\mathbf{R}_r$ is the r-th order residue matrix and satisfies Eq. (4); $\boldsymbol{\varphi}_r$ is the r-th order modal vector; $\boldsymbol{\Gamma}_r$ is the modal participation factors or modal amplitude vector for the r-th order mode.

Taking Eq. (2) into Eqs. (1), (7) would be obtained.

$$\begin{aligned}\mathbf{G}_{yy}(j\omega) &\approx \sum_{r=1}^{N}\sum_{k=1}^{N}\left(\frac{\mathbf{R}_r}{j\omega-\lambda_r} + \frac{\mathbf{R}_r^*}{j\omega-\lambda_r^*}\right)^*\mathbf{G}_{xx}(j\omega)\left(\frac{\mathbf{R}_k}{j\omega-\lambda_k} + \frac{\mathbf{R}_k^*}{j\omega-\lambda_k^*}\right)^{\mathrm{T}}\\ &= \mathbf{G}_{xx}(j\omega)\sum_{r=1}^{N}\sum_{k=1}^{N}\left(\frac{\mathbf{R}_r^*}{-j\omega-\lambda_r^*} + \frac{\mathbf{R}_r}{-j\omega-\lambda_r}\right)\left(\frac{\mathbf{R}_k^T}{j\omega-\lambda_k} + \frac{\mathbf{R}_k^H}{j\omega-\lambda_k^*}\right)^T\end{aligned} \tag{7}$$

If $f(t)$ is ambient excitation, then $\mathbf{G}_{xx}(j\omega)$ is a diagonal matrix whose entries are constant. According to Heaviside partial-fraction expansion theorems, Eq. (7) can be derived as Eq. (8).

$$\mathbf{G}_{yy}(j\omega) \approx \sum_{r=1}^{n}\frac{\mathbf{A}_r}{j\omega-\lambda_r} + \frac{\mathbf{A}_r^H}{-j\omega-\lambda_r^*} + \frac{\mathbf{A}_r^T}{-j\omega-\lambda_r} + \frac{\mathbf{A}_r^*}{j\omega-\lambda_r^*} \tag{8}$$

with

$$\mathbf{A}_r = \sum_{k=1}^{n}\left(\frac{\mathbf{R}_k}{-\lambda_r-\lambda_k} + \frac{\mathbf{R}_k^*}{-\lambda_r-\lambda_k^*}\right)\mathbf{G}_{xx}\mathbf{R}_r^T \tag{9}$$

$$\lambda_r = -\zeta_r\omega_r + i\omega_{dr} = -\zeta_r\omega_r + i\omega_r \tag{10}$$

where $\mathbf{A}_r$ is the r-th order residual matrix of the power spectral density matrix and satisfies Eq. (9), which also has the conjugate form $\mathbf{A}_r^*$.In general situation, the structural damping is small, the r-th order pole λ_r can be represented as Eq. (10) according to Eq. (3). If there are no close natural frequencies, the value of the denominator with $k = r$ is much smaller than the one with $k \neq r$. Then Eq. (9) can be simplified as Eq. (11).

$$\mathbf{A}_r = \left(\frac{\mathbf{R}_r}{2(\zeta_r\omega_r-i\omega_r)} + \frac{\mathbf{R}_r^*}{2\zeta_r\omega_r}\right)\mathbf{G}_{xx}\mathbf{R}_r^T = \frac{\mathbf{R}_r^*\mathbf{G}_{xx}\mathbf{R}_r^T}{2\zeta_r\omega_r} = \beta_r\boldsymbol{\varphi}_r^*\boldsymbol{\varphi}_r^T \tag{11}$$

with

$$\beta_r = (\boldsymbol{\Gamma}_r^H\mathbf{G}_{xx}\boldsymbol{\Gamma}_r)/2\zeta_r\omega_r \tag{12}$$

Due to β_r is a scalar, the Eq. (13) can be obtained.

$$\mathbf{G}_{yy}(j\omega) \approx \sum_{r=1}^{n}\frac{\beta_r\boldsymbol{\varphi}_r^*\boldsymbol{\varphi}_r^T}{j\omega-\lambda_r} + \frac{\beta_r\boldsymbol{\varphi}_r^*\boldsymbol{\varphi}_r^T}{-j\omega-\lambda_r^*} + \frac{\beta_r\boldsymbol{\varphi}_r\boldsymbol{\varphi}_r^H}{-j\omega-\lambda_r} + \frac{\beta_r\boldsymbol{\varphi}_r\boldsymbol{\varphi}_r^{\mathrm{H}}}{j\omega-\lambda_r^*} \tag{13}$$

In the narrowband of the r-th order, the values of the first two denominators in Eq. (13) satisfies Eq. (14), which are much smaller than the ones of the last two denominators that meets Eq. (15). Therefore, the last two parts can be omitted, which gives another expression of Eq. (13) as Eq. (16).

$$-j\omega-\lambda_r^* = (j\omega-\lambda_r)^* \simeq \zeta_r\omega_r \tag{14}$$

$$j\omega-\lambda_r^* = (-j\omega-\lambda_r)^* \simeq \zeta_r\omega_r + 2j\omega_r \tag{15}$$

$$\mathbf{G}_{yy}(j\omega) \approx \sum_{r=1}^{N}\left(\frac{\beta_r\boldsymbol{\varphi}_r^*\boldsymbol{\varphi}_r^T}{j\omega-\lambda_r} + \frac{\beta_r\boldsymbol{\varphi}_r^*\boldsymbol{\varphi}_r^T}{-j\omega-\lambda_r^*}\right) = \Phi^* diag\left(2\mathrm{Re}\left(\frac{\beta_r}{j\omega-\lambda_r}\right)\right)\Phi^T \tag{16}$$

where $\Phi = [\boldsymbol{\varphi}_1\boldsymbol{\varphi}_2\cdots\boldsymbol{\varphi}_n]$ is the modal matrix. The power spectral density matrix of the r-th order mode can be represented as Eq. (17).

$$\begin{aligned}\mathbf{G}_{yy}(j\omega)_{\omega\to\omega_r} &\approx \boldsymbol{\varphi}_r^*\left\{diag\left(2\mathrm{Re}\left(\frac{\beta_r}{j\omega-\lambda_r}\right)\right)\right\}\boldsymbol{\varphi}_r^T\\ &= \boldsymbol{\varphi}_r^*\left\{diag\left(\frac{2\beta_r\zeta_r\omega_r}{(\zeta_r\omega_r)^2 + (\omega-\omega_{dr})^2}\right)\right\}\boldsymbol{\varphi}_r^T\end{aligned} \tag{17}$$

From Eq. (17), it is obvious that the formulation form is the singular value decomposition. There is only one mode to contribute the most of power spectral energy when the structural frequency is close to some natural frequency. The modal vector corresponding to this natural frequency is $\boldsymbol{\varphi}_r$.

Therefore, when the measurement is obtained, the power spectral density can be generated. The frequency and the corresponding modal parameters at the point of the peak value of the power spectral density is the structural natural frequency.

3. Modified frequency domain decomposition

When there exist closely spaced modes, for example, r-th order mode and $r + 1$-th order mode are close, then Eq. (9) would be simplified as Eq. (18), which is different from Eq. (11).

$$\begin{aligned}\mathbf{A}_r &= \left(\frac{\mathbf{R}_{r+1}}{2(\zeta_r\omega_r-j\omega_r)} + \frac{\mathbf{R}_{r+1}^*}{2\zeta_r\omega_r}\right)\mathbf{G}_{xx}\mathbf{R}_r^T + \left(\frac{\mathbf{R}_{r+1}}{(\zeta_r\omega_r-j\omega_r) + (\zeta_{r+1}\omega_{r+1}-j\omega_{r+1})}\right.\\ &\quad\left. + \frac{\mathbf{R}_{r+1}^*}{(\zeta_r\omega_r-j\omega_r) + (\zeta_{r+1}\omega_{r+1} + j\omega_{r+1})}\right)\mathbf{G}_{xx}\mathbf{R}_r^T\\ &= \frac{\mathbf{R}_r^*\mathbf{G}_{xx}\mathbf{R}_r^T}{2\zeta_r\omega_r} + \frac{\mathbf{R}_{r+1}^*\mathbf{G}_{xx}\mathbf{R}_r^T}{2(\zeta_r + \zeta_{r+1})\omega_r} = \beta_r\boldsymbol{\varphi}_r^*\boldsymbol{\varphi}_r^T + \alpha_r\boldsymbol{\varphi}_{r+1}^*\boldsymbol{\varphi}_r^T\end{aligned} \tag{18}$$

Due to $\boldsymbol{\Gamma}_r = u_r\boldsymbol{\varphi}_r$ and the orthogonality of the modal vector, $\alpha_r = (\boldsymbol{\Gamma}_{r+1}^H\mathbf{G}_{xx}\boldsymbol{\Gamma}_r)/2\zeta_r\omega_r$ would be close to zero, which causes that the second part can be omitted. Therefore, when the r-th order mode and $r + 1$-th order mode are close, the power spectral density matrix can be expressed as Eq. (19).