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Nonlinear Optimal Control in Catalytic Process via Stable Manifold Method ⋆

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Abstract: This paper proposes an optimal feedback control design for the nonlinear partial differential equation of a catalytic rod in a reactor via stable manifold method. Stable manifold method provides numerical stabilizing solutions of Hamilton-Jacobi equations in nonlinear optimal control theory. We apply this method to a reduced order system obtained from the proper orthogonal decomposition and Galerkin projection. The feasibility of the design is demonstrated by a numerical example.

Keywords: optimal control, distributed-parameter system, nonlinear systems, proper orthogonal decomposition, stable manifold method

1. INTRODUCTION

Important control objects with complexity in engineering, e.g., thermal process, chemical process, fluids, and flexible structures can be described by partial differential equations (PDEs). Such systems of PDEs generally possess nonlinearity and an infinite degree of freedoms; therefore, it is quite difficult to apply systematic control designs to them. As a solution of overcoming the difficulty, this paper proposes an optimal regulator design for controlling heat distributions of a catalytic rod in a reactor in terms of *POD-Galerkin method* (Holmes et al., 1998) and *stable manifold method* (Sakamoto and van der Schaft, 2008).

The proper orthogonal decomposition (POD) basis is a set of basis functions obtained from time responses of the systems. POD-Galerkin method is a model reduction technique for deriving approximate lower dimensional systems of ordinary differential equations described by a POD basis from systems of PDEs, which has been widely applied to fluid mechanics (please see, e.g., Holmes et al. (1998); Kunisch et al. (2004)).

Stable manifold method is an efficient numerical solver for calculating stabilizing solutions of Hamilton-Jacobi equations (HJEs) in nonlinear optimal control problems. HJEs are first-order partial differential equations, and it is difficult to analytically solve them except for some special cases. Stable manifold method has been applied to various practical nonlinear systems (Sakamoto, 2012).

The procedure of the control design presented in this paper is as follows. We first derive a reduced model from a control system of PDEs by POD-Galerkin method. Next, we formulate a nonlinear optimal control problem for the reduced model with a desired cost function. Finally, we find stabilizing solutions of the HJE of the problem by stable manifold method. The solutions can determine the nonlinear gain of the optimal feedback controller.

In this paper, we consider a thermal control of a catalytic rod in a reactor as an illustrative example of the optimal control. The thermal controls have been studied (Christofides, 2001; Ray, 1980), in particular, there is also a result via POD-Galerkin method (Varshney, 2009). However, these are studied from mainly the viewpoint of stabilization. Hence, we attempt to incorporate optimality into the control, which has not been sufficiently discusses yet.

In the catalytic system, the origin that means a uniform distribution of temperatures is an unstable equilibrium point, and a stable equilibrium point is some distance away from the origin. We show that the nonlinear regulator can achieve a state transition between these equilibria with less control inputs in the sense of optimal control. This

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control is expected to develop ecological process controls for complex systems described by PDEs.

2. SUMMARY OF MODEL REDUCTION VIA POD-GALERKIN METHOD

2.1 Notations

Let X be a real Hilbert Space. The inner product and the norm in X are denoted by $(\cdot,\cdot)_X$, $\|\cdot\|_X$, respectively. X' denotes the dual space of X, and $(\cdot,\cdot)_{X'\times X}$ denotes the dual pairing. We define the space $L^2(0,T;X)$ for $0 < T \leq \infty$

$$L^2(0,T;X) = \left\{ w(t) \in X \text{ a.e. } t \in (0,T) \;\middle|\; \int_0^T \|w(t)\|_X^2 dt < \infty \right\}, \tag{1}$$

the space

$$W(0,T;X) = \{\phi \in L^2(0,T;X) \mid \phi_t \in L^2(0,T;X')\}, \tag{2}$$

and the set

$$W_{loc}(0,\infty;X) := \bigcap_{T>0} W(0,T;X). \tag{3}$$

Let V and H be real separable Hilbert spaces such that $V \subset H$ and V dense in H. We assume that the injection of $V \subset H$ is compact. We consider a symmetric bilinear continuous form $a : V \times V \to \mathbb{R}$ that is coercive, i.e., there exists a constant $\kappa > 0$ such that $a(v,v) \geq \kappa\|\phi\|_V^2$ for all $v \in V$. Let $N : V \to V'$ be a nonlinear continuous operator mapping satisfied $N(0) = 0$ and it's Frèchet derivative $N'(0) = 0$.

2.2 Control systems

We consider the nonlinear evolution equation

$$\frac{d}{dt}(y(t),\varphi)_H + a(y(t),\varphi) + (N(y(t)),\varphi)_{V',V} = (Bu(t),\varphi)_{V',V} \tag{4a}$$

for all $\varphi \in V$ with the initial condition

$$y(0) = y_0 \in H, \tag{4b}$$

where $y(t) \in V$ is the state, $u(t) \in \mathbb{R}^m$ is the control input and $B : \mathbb{R}^m \to V'$ is a continuous linear operator.

2.3 Proper orthogonal decomposition

Let us consider the set of functions

$$U = \{y_k : y_k \in X\}_{k=1}^n, \tag{5}$$

where X is a separable Hilbert space and n is the number of elements in U. Practically, U is derived from a free response $y(t)$ of the system (4a, 4b) with $u(t) = 0$, i.e., $y_k := y(t_k)$ at each discrete time t_k for $k = 1, \cdots, n$, which is called a *snapshot set*. We define an average of the image $f(U)$ by

$$\langle f(y_k)\rangle_k = \frac{1}{n}\sum_{k=1}^n f(y_k), \tag{6}$$

where f is a map $f : X \to \mathbb{R}$ or X.

We shall consider an optimal orthogonal basis for describing the set U. We first define a function φ as the optimal solution maximizing the objective function

$$\max_{\varphi\in X} \frac{\langle (y_k,\varphi)^2\rangle_k}{\|\varphi\|^2}. \tag{7}$$

The optimal solution φ is the most parallel function to the set U in the sense of mean square.

From the necessary condition of the optimality, we can obtain the following relation (Holmes et al., 1998):

$$\mathcal{R}\varphi = \lambda\varphi, \tag{8}$$

where $\lambda \in \mathbb{R}$ is non-negative and $\mathcal{R} : X \to X$ is defined by

$$\mathcal{R}\varphi = \langle (y_k,\varphi)y_k\rangle_k. \tag{9}$$

Actually, $\mathcal{R}$ is a compact operator, and there exist an optimal solution of (7). The maximum value of (7) corresponds with the maximum eigenvalue of (8). By Hilbert-Schmidt theorem, there exist an orthonormal basis consisting of eigenvectors in (8). Then, there exist eigenvalues $\lambda_1 \geq \lambda_2 \cdots \geq 0$. The orthogonal basis obtained from this procedure is called a *POD basis*.

Remark 1. We don't directly solve the eigenvalue problem of (8) to get a POD basis, but another eigenvalue problem of a certain matrix $\mathcal{K}$, because the former is difficult to solve it (Kunisch et al., 2004). The matrix $\mathcal{K} \in \mathbb{R}^{n\times n}$ is defined by

$$\mathcal{K} = \frac{1}{n}\mathcal{U}^*\mathcal{U}, \tag{10}$$

where $\mathcal{U} : \mathbb{R}^n \to X$ is the linear bounded operator such that

$$\mathcal{U}v = \sum_i v_i y_i \tag{11}$$

for $v = [v_1, \cdots, v_n]^T \in \mathbb{R}^n$, and $\mathcal{U}^*$ is the adjoint operator of $\mathcal{U}$, i.e.,

$$\mathcal{U}^* w = [(w,y_1), \cdots, (w,y_n)]^T \tag{12}$$

for $w \in X$. Then, the map $\mathcal{R} : X \to X$ can be considered as

$$\mathcal{R} = \frac{1}{n}\mathcal{U}\mathcal{U}^*, \tag{13}$$

where $\mathcal{R}$ is bounded, non-negative and self-adjoint operator.

2.4 Galerkin projection and reduced order models

A POD basis can efficiently represent the energy of free responses used for making snapshot sets (Holmes et al., 1998). The ratio of an energy described by primal basis functions up to r-th to a total energy can be expressed as

$$\mathcal{E}(r) := \sum_{j=1}^r \lambda_j \Big/ \sum_{j=1}^n \lambda_j. \tag{14}$$

By using Galerkin projection with POD basis, we can derive a finite-dimensional reduced system from an infinite-dimensional system (4a, 4b). We first determine a reduced-order r to sufficiently describe the response, namely $\mathcal{E}(r) \approx 1$. Then, the signal y can be written as follows:

$$y(t) = \sum_{i=1}^r a_i(t)\varphi_i. \tag{15}$$

By substituting (15) into the left-side of (4a) and take the inner product with each basis φ_j for $j = 1, \cdots, r$, the finite-dimensional reduced system can be written as follows:

$$\dot{a} = A_r a + N_r(a) + B_r u, \tag{16}$$