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Least squares 2D bi-cubic spline approximation: Theory and applications

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Abstract. Smooth surface approximation plays an important role in many applications. As an extension of the 2D bi-cubic spline interpolation, we propose the least squares 2D bi-cubic spline approximation (LS-BICSA). LS-BICSA, generally applicable to a set of irregular data, is formulated by the pure bi-cubic spline functions. The method is flexible because the user can impose the appropriate continuity and differentiability conditions at the boundary points using the available constrained least squares theory. Two scenarios having two and four knots at the boundary points are described in this paper. Two examples are presented to illustrate the theory. The first example approximates a known mathematical function on an irregular grid. The second example uses a real data set to approximate the geoid height. LS-BICSA can thus provide reliable results for many geoscience applications where it is required to approximate the function values using a smooth spline surface in the least squares sense.

Keywords*:* Least squares 2D bi-cubic spline approximation (LS-BICSA); Continuity and differentiability conditions; Constrained least squares; Geoid height approximation

1 Introduction

In many science and engineering applications, fitting a curve in one-dimensional (1D) space or a surface in two-dimensional (2D) space to a set of randomly scattered data points is a commonly encountered problem. To estimate the unknown function values of known data points, two methods are usually used: interpolation and approximation. Estimating the unknown coefficients of a polynomial passing through *m* points (known as knots) is referred to as interpolation. Approximation however requires that the polynomial gets sufficiently close to the knots [1]. The nature of the problem at hand usually determines to use either interpolation or approximation. A