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Angular-rate sensing by mode splitting in a Whispering-gallery-mode optical microresonator

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A B S T R A C T

Angular- rate sensing can be achieved by monitoring the shift in or splitting of a resonant frequency. The mode-shift mechanism, described by the Sagnac effect, treats backscattering as a noise source in a traditional resonant optic gyroscope. While in an ultrahigh-Q resonator, backscattering can create mode splitting. Mode splitting as a self-reference sensing scheme can solve problems such as temperature vibration, which leads to a higher resolution. We propose the mode-splitting mechanism for angular-rate sensing. It is found that the angular rate can be derived from the amount of splitting instead of the frequency difference between the clockwise and counterclockwise waves. The theoretical highest resolution can reach 0.0614°/s. To realize enhanced performance, the issues that may affect the measurement are discussed.

1. Introduction

Owing to the advantages offered by high quality factor (Q factor) and a silica-based substrate, a whispering-gallery-mode (WGM) microresonator [1,2] can be developed as the key component of the sensor this approach has attracted widespread attention. Scattering-induced mode splitting in WGM microresonator has been demonstrated both theoretically and experimentally. Each traveling-wave mode in a microresonator possesses a twofold degeneracy resulting from two counter-propagating directions: clockwise (CW) and counterclockwise (CCW). A subwavelength scatterer in the optical path of a traveling-wave mode has the ability to scatter a fraction of energy into the opposite propagating mode and thus create coupling of the traveling modes. This coupling lifts the degeneracy and gives rise to two standing-wave modes [the symmetric mode (SM) and asymmetric mode (ASM)] that are manifested by the doublet in the transmission spectrum [3–5]. The mechanisms of splitting by distributed scattering in a static microresonator were theoretically investigated and were applied in the label-free detection of nanoparticles at the single-molecule resolution [6–8].

The temperature shift has great influence on angular rate sensing in a resonant optic gyroscope (ROG) [9,10]. While Mode splitting as a self-reference sensing scheme [11] is more immune to disturbance from the environment such as temperature vibration. It can provide superior noise suppression and enable accurate measurements [12]. Owing to Sagnac effect, the degeneracy can also be lifted by rotating the resonator, which causes the CW and CCW modes to exhibit different round-trip times. This shows great potential for achieving high-sensitivity angular-rate sensing for the highly dispersive property along with a low absorption at the resonance.

In this paper, considering both Sagnac and mode splitting effects, we present, to the best of our knowledge, the first analysis of single-particle-induced mode splitting in a WGM microresonator for detecting the angular rate.

2. Theoretical model describing the mode splitting in a rotational WGM microresonator

Ideal coupling does not exist in a pure circular microresonator. Backscattering can be created when spherical symmetry is broken owing to minor contamination on the resonator surface, structural defects, or material inhomogeneity [5]. This type of mode splitting is called intrinsic splitting. While we focus on induced mode splitting situation in which the coupling is induced by a scatterer. The scatterer mediates a coherent coupling of the traveling modes and causes their consequent mode splitting. The cross-coupling can redistribute energy between the split modes and couple the confined WGMs to radiation modes, inducing scattering loss and thus line-width broadening [3]. Fiber taper coupling serves as a method to excite the resonator, as shown in Fig. 1.

Given a circular resonator of radius R with an angular rate Ω, the result for the rotation-induced phase deviation between the CW and CCW waves during a round trip is [13,14]

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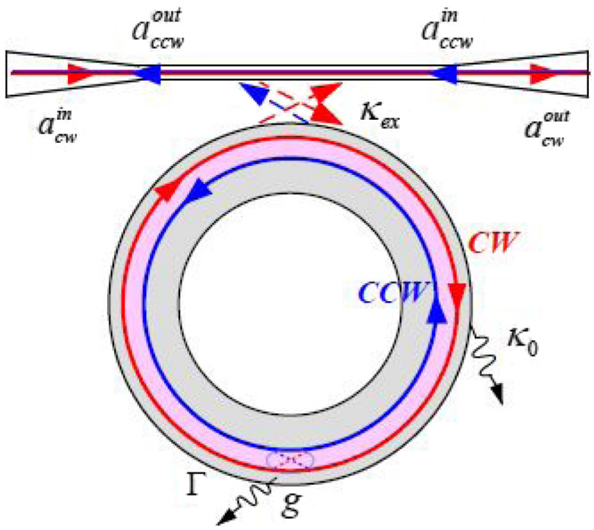

Fig. 1. Illustration of the coupled microresonator–scatterer system. κ_0: intrinsic damping rate, κ_{ex}: resonator-taper coupling rate, ξ: round-trip energy gain of the active medium, g: internal coupling coefficient of the light scattered into the resonator, Γ: additional damping rate due to scattering loss to the environment. a_{cw}^{in} and a_{cw}^{out} are the CW input and output fields of the waveguide. a_{ccw}^{out} is the reflected field.

$$\Delta\varphi = \frac{8\pi^2R^2}{\lambda c}\Omega \tag{1}$$

where λ is the resonant wavelength, c is the speed of light. Then the angular-frequency deviation of the CW and CCW waves can be expressed as

$$\Delta\omega_{sub} = \frac{\Delta\varphi}{\tau_R} = \frac{\Delta\varphi}{\frac{2\pi R n_{eff}}{c}} = \frac{2\omega_c R}{n_{eff}\, c}\Omega \tag{2}$$

where τ_R denotes a round-trip time, and $\tau_R = 2\pi R n_{eff}/c$, n_{eff} is the effective refractive index, ω_c indicates the resonance frequency of the initially degenerate mode, and $\omega_c = (2\pi c)/\lambda$. The CW or CCW frequency deviation per unit time is $\Delta\omega_t = \frac{1}{2}\Delta\omega_{sub}$. It is found that a microresonator with an ultrahigh Q enters a regime where the scattering of light into the degenerate pair of CW and CCW modes is the dominant scattering process [4]. Assuming that the resonator works in a slowly varying envelope approximation, the traveling modes of a fiber-coupled resonator system can be described with a coupled harmonic-oscillator model. In this section, we assume single-polarization operation for simplicity. Considering a rotation in the CCW direction, we modify the rate equations that describe the coupling of the two traveling modes via a scatterer by incorporating the angular-frequency deviation from the rotation and write them in a matrix as

$$\frac{d}{dt}\begin{bmatrix} a_{cw} \\ a_{ccw} \end{bmatrix} = \begin{bmatrix} -i(\omega_c + g + \Delta\omega_t) - \frac{\beta}{2} & -\left(ig + \frac{\Gamma}{2}\right) \\ -\left(ig + \frac{\Gamma}{2}\right) & -i(\omega_c + g - \Delta\omega_t) - \frac{\beta}{2} \end{bmatrix}\begin{bmatrix} a_{cw} \\ a_{ccw} \end{bmatrix} - \begin{bmatrix} \sqrt{\kappa_{ex}} \\ 0 \end{bmatrix} a_{cw}^{in}, \tag{3}$$

where a_{cw} and a_{ccw} are intracavity field amplitudes of the CW and CCW modes. $\beta = \Gamma + \kappa_0 + \kappa_{ex} - \xi$ is defined as the effective damping rate. The damping loss induced by the scatterer is made up of two parts. One contributes to the intrinsic damping rate, as seen in the rate β; the other contributes to the scattering loss during mode coupling, as seen in the complex internal coupling coefficient $-(ig + \Gamma/2)$. We call the former damping loss I, and the latter is called damping loss II. The input–output relationship of the fiber-coupled resonator system is $a_{cw}^{out} = a_{cw}^{in} + \sqrt{\kappa_{ex}}\cdot a_{cw}$.

To obtain the locations of the split modes, we transform Eq. (3) into a coordinate system that consists of the eigenvectors of its characteristic matrix. This results in an orthogonal basis for the solution

$$\frac{d}{dt}\begin{bmatrix} a_{SM} \\ a_{ASM} \end{bmatrix} = \begin{bmatrix} \lambda_{SM} & 0 \\ 0 & \lambda_{ASM} \end{bmatrix}\begin{bmatrix} a_{SM} \\ a_{ASM} \end{bmatrix} + \begin{bmatrix} \frac{\sqrt{\kappa_{ex}}\,(ig + \Gamma/2)}{2\sqrt{(ig + \Gamma/2)^2 - \Delta\omega_t^2}} \\ \frac{-\sqrt{\kappa_{ex}}\,(ig + \Gamma/2)}{2\sqrt{(ig + \Gamma/2)^2 - \Delta\omega_t^2}} \end{bmatrix} a_{cw}^{in}. \tag{4}$$

The eigenvalues of the characteristic matrix can be expressed as

$$\lambda_{SM} = -i(\omega_c + g) - \frac{\beta}{2} + \sqrt{\left(ig + \frac{\Gamma}{2}\right)^2 - \Delta\omega_t^2}, \tag{5}$$

$$\lambda_{ASM} = -i(\omega_c + g) - \frac{\beta}{2} - \sqrt{\left(ig + \frac{\Gamma}{2}\right)^2 - \Delta\omega_t^2}. \tag{6}$$

The matrix in Eq. (4) has no nonzero off-diagonal terms, which means that the a_{SM} and a_{ASM} modes are orthogonal to each other. In the presence of a rotation, the eigenmodes still behave like standing waves, which is similar to the static case. The imaginary parts of the eigenvalues imply the resonance frequencies, which can be expressed as

$$\omega_{SM} = -\mathrm{Im}(\lambda_{SM}) = \omega_c + g + \frac{\Gamma g}{\sqrt{2\left(\gamma + \sqrt{\gamma^2 + \Gamma^2 g^2}\right)}} \tag{7}$$

$$\omega_{ASM} = -\mathrm{Im}(\lambda_{ASM}) = \omega_c + g - \frac{\Gamma g}{\sqrt{2\left(\gamma + \sqrt{\gamma^2 + \Gamma^2 g^2}\right)}} \tag{8}$$

where $\gamma = \Gamma^2/4 - g^2 - \Delta\omega_t^2$, and Im{…} implies that the imaginary part is taken. ω_{ASM} and ω_{SM} represent the eigenfrequencies of the ASM and SM, respectively. The amount of splitting can be calculated from $\delta = |\omega_{ASM} - \omega_{SM}|$. Hence when temperature changes, though both modes (SM and ASM) shift in the same direction, the deviation can be counteracted when the amount of splitting is utilized to measure the angular velocity. A rotation shifts the split modes to opposite directions. Using Eqs. (7) and (8), we obtain an explicit expression for the amount of splitting δ versus Ω

$$\delta = \sqrt{\delta_0^2 + \frac{\Omega^2\left(\frac{2\omega_c R}{n_{eff}\, c}\right)^2}{\left(1 + \frac{\Gamma^2}{\delta^2}\right)}} \tag{9}$$

or

$$\Omega = \frac{n_{eff}\, c}{2\omega_c R}\sqrt{(\delta^2 - \delta_0^2)\left(1 + \frac{\Gamma^2}{\delta^2}\right)} = \frac{n_{eff}\lambda}{4\pi R}\sqrt{(\delta^2 - \delta_0^2)\left(1 + \frac{\Gamma^2}{\delta^2}\right)} \tag{10}$$

where $\delta_0 = |2g|$ is the static (nonrotating) amount of splitting. It has been proved that for a certain resonator this static amount of splitting is also related with refractive index and size of the particle but independent of its position [12]. If we assume that the resonator works in the absorption-limited regime ($\omega_c/Q >> 2\Gamma$), we ignore damping loss II. Therefore, the eigenfrequencies become

$$\omega_{SM} = \omega_c + g - \sqrt{g^2 + \Delta\omega_t^2} \tag{11}$$

$$\omega_{ASM} = \omega_c + g + \sqrt{g^2 + \Delta\omega_t^2} \tag{12}$$

In such a case, the expressions of the eigenfrequencies versus the angular rate change to hyperbolas. Eq. (9) becomes

$$\delta = \sqrt{\delta_0^2 + \Omega^2\left(\frac{2\omega_c R}{n_{eff}\, c}\right)^2} \tag{13}$$

The Sagnac effect presents the expression of the resonant frequencies of the two traveling modes (CW, CCW) versus the angular rate, which exhibits a linear relationship. Fig. 2 shows the calculated results of the resonance frequencies of the two standing modes (SM, ASM) versus the angular rate and the amount of splitting versus the angular