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Optimization of pulse width for frequency measurement by the method of rational approximations principle

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ABSTRACT

The purpose of any measurement is to obtain the required approximation to the measurand. In particular, when measuring time-frequency parameters, traditional methods require more time for measurement, if the frequency value has to be measured more accurately. It is proved that the principle of rational approximations is a method for estimating an unknown frequency, which is very precise, and requires a relatively short time for measurement. Its main properties are: (i) insensitivity to most known sources of uncertainty; (ii) the time required for the measurement is reduced when measuring higher frequencies; (iii) the accuracy of measurements depends on the stability of the reference frequency standard. The recently investigated property of the principle of rational approximations is the dependence of the measurement error on the width of the pulse in the signals used for measurement. In particular, if the pulse width is smaller, the error in the measurement process decreases more rapidly. However, a generally recognized model of analysing how short the pulse width should be is not yet created. In the present paper, a formalism has been developed to determine the optimum pulse width in order to minimize both the magnitude of the error and the measurement time. Variation of this parameter permits to obtain the best approximation to the measurand from the very first moments of measurement. The minimum time required for the measurement was thoroughly analysed.

1. Introduction

When measuring time-frequency parameters, the measurement goal is to obtain the best approximation to the unknown actual frequency value, at an acceptable time. The period of the signal depends of its frequency, in other words, if signal frequency increases, the duration of signal period decreases. The measurement methods used in modern commercial frequency meters [1] require more time for measuring the frequency value (f_x) if the highest accuracy is required. It is well known that for any frequency measurement, $M_t > T_x$, where T_x denotes the period corresponding to f_x, and the measurement time is named as M_t. Also during M_t, deviations of unknown frequency measurement are reduced, until the measurement error starts to be acceptable for application's accuracy requirements. The most significant difference between the principle of rational approximations and other methods [1–3] of measuring the frequency described in the literature is that this method is insensitive to jitter [4], and the accuracy of the measurement is determined by the reference standard only. The proposed applications of this principle of measurement are quite diverse [5–11], which motivates further investigation of its properties. In this study, the previous theoretical work [12] is expanded, and the mathematical model for the optimization of pulse width is analysed, with the final goal to measure the frequency in any range with the best accuracy during the shortest time.

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2. Fast frequency measurements using the principle of rational approximations

This method is based on number theory, in particular a property of rational numbers as the mediant fractions [13]. Frequency measurement is done by comparing two signals, a reference (S_0) whose frequency value is known (f_0) and a signal to measure (S_x) with unknown frequency (f_x). The periods of both unknown and reference signals are denoted by T_x, and T_0 respectively. Both signals must have the same pulse width (τ), and $\tau < T_0/2$ (or $f_0 \geqslant 1/2\tau$ [14]). As it was shown in [12], each pulse (in each signal) has a particular time for starting and ending. From this statement, when using this method of frequency measurement, during the "comparison of signals", a search for overlapped pulses is done. Also in [12] it was shown, that overlapped pulses are detected when input signals are "logically multiplied". If there is a temporary overlapping of the pulses, then $S_x S_0 = 1$, otherwise $S_x S_0 = 0$. For this reason, when this method is implemented in experimental prototypes, AND-gate is the nucleus of coincidence detectors [12,13].

Heretofore, applications of the principle of rational approximations follow the criterion of $\tau < T_0/2$. Since $f_0 > f_x$, the condition $\tau < T_0/2$ allows to avoid one pulse from any signal to overlap two pulses from the other signal. In case the last condition is not kept, the multiple overlapping of one pulse leads to unpredicted results for this method, because the mediant condition could be affected. As a consequence, accurate measurements are not guaranteed.

When signals are compared, a pulse train of coincident pulses (S_x & S_0) is generated (for comparison, a coincidence detection circuit is required, an example of such a circuit is presented in [15]). The process of measuring the frequency starts at the moment when the first coincidence of the pulses exists, and the counting of pulses in S_x,S_0 begins simultaneously. The number of pulses in S_x,S_0 is denoted by P_n,Q_n, where n is the number of coincidence (Fig. 1a).

The duration of pulse overlapping is known as the coincidence time (t_{0x}). From Fig. 1a, two types of coincidences can be observed, perfect ($t_{0x} = \tau$) and partial coincidences ($t_{0x} < \tau$). When measuring, specially when comparing signals, it does not matter what kind of coincidence is detected. As long as there is a coincidence, the corresponding $n-$fraction is registered (this property is illustrated in Fig. 1). This fact is another insufficiently known source of uncertainty in the measurement. Proceeding from this, the initial approximations to the measurand can have the highest error.

As soon as the pulses are counted, after any coincidence, the value of the unknown frequency is approximated by:

$$f_x = f_0 \frac{P_n}{Q_n}, \tag{1}$$

when $n \geqslant 1$, or by the sum of all numerators and denominators that form the mediant fraction m [12,13]

$$f_x = f_0 \frac{\sum_m P_n}{\sum_m Q_n}. \tag{2}$$

The measurement time (M_t) is given by

$$M_t = Q_n T_0 = \frac{Q_n}{f_0}, \tag{3}$$

and the relative error (β) in frequency measurement process is calculated by

$$\beta = \left| \frac{f_x - \frac{P_n}{Q_n} f_0}{f_x} \right| \tag{4}$$

or

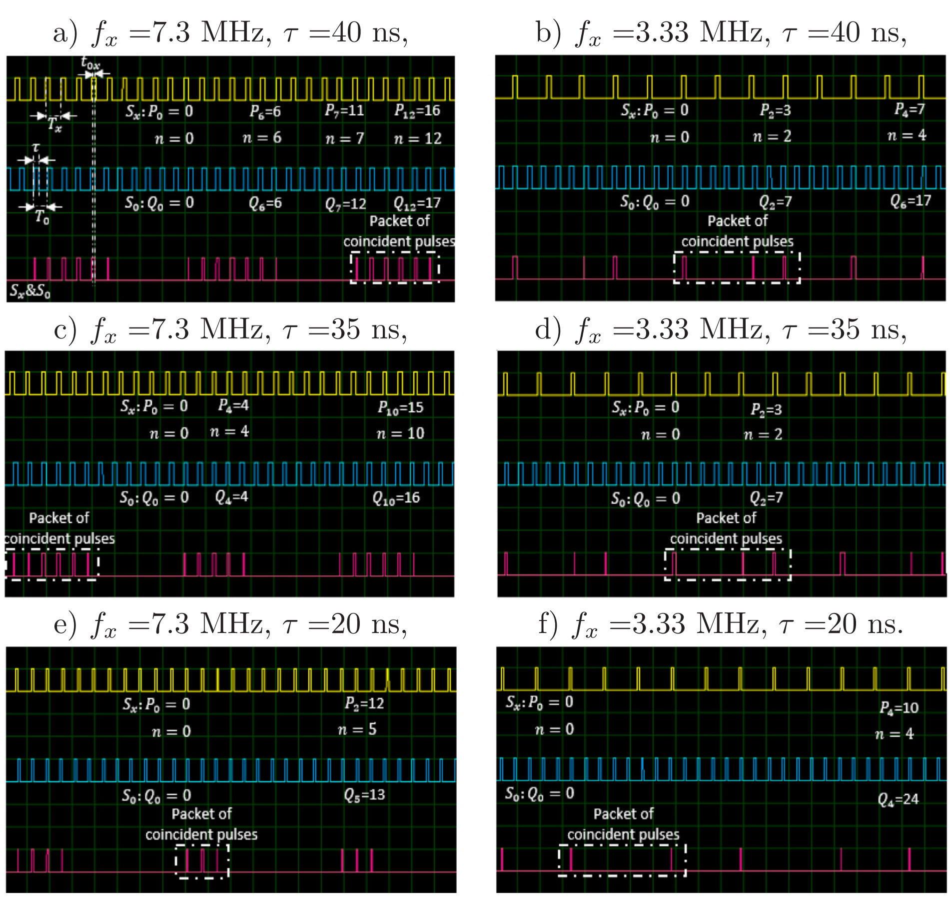

Fig. 1. Signal comparison process in the rational approximations principle, using f_0 = 8 MHz.