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Measurement and mathematical model of convexo-concave Novikov gear mesh

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ABSTRACT

This paper presents a novel measurement method and a mathematical model of external and internal convexo-concave Novikov gear mesh with a single line of action. Parametric equations of tooth surfaces as well as normal units were obtained. The mathematical model was applied in tooth contact analysis for sample gear pairs. The abovementioned analysis involved obtaining the line of action, paths of contact point on tooth surfaces and contact patterns. In addition, an experiment aiming at the verification of adopted mathematical model was carried out. The results confirm that the presented model can be used to simulate the phenomena at work in real gear mesh. The study and its verification proved good consistency of tooth contact analysis results based on a developed mathematical model with experimental data. Mean deviation between theoretical and experimental area of contact was 0.91% for external and 1.37% for internal gearing.

1. Introduction

A tooth profile commonly used in gear transmissions is an involute one, where in case of external gearing a convex pinion tooth flank meshes with a convex gear tooth flank. The character of this contact is particularly adverse as regards surface strength. Numerous studies on the possibility of increasing surface strength of gears have concentrated on extensive investigations in which numerical and empirical models were published [1–3]. Lin et al. [4]; Dudas and Bodzas [5]; Bodzas and Dudas [6] presented mathematical descriptions, geometry analysis and modelling of gear teeth fabricated by rapid prototyping. Chen et al. [7] presented an analysis of a pair of conjugate surfaces and their instantaneous contact-line. In this paper, basic principles of conjugation are developed. The present investigations lay foundations for transforming the principle of surface conjugation in practice. Litvin et al. [8] developed finite element models of a new type of Novikov-Wildhaber helical gear drive. They focused on design, generation, tooth contact analysis (TCA) and stress analysis. Ye and Ye [9] developed a new method for seeking optimum gear tooth profiles, as illustrated by an example of the line-contact helical gearing. Chen et al. [10] proposed an approach for computerized simulation of meshing and contact of misaligned hyperboloid-type normal circular-arc gears. Using the moving frame method, variations and errors were quantified and the influence of misalignment errors on the shift of the bearing contact was investigated. Deng et al. [11] measured and evaluated tooth surface precision. According to Korotkin and Gazzaev [12], the improvement of reference profiles and methods for calculating teeth at bending is a topical problem, the solution of which will significantly expand the scope of the use of a progressive Novikov gearing. One of the methods of increasing a gear's surface load capacity is to use special tooth profiles in which the concave flank meshes with the convex flank of the mating gear. One of these profiles is the Novikov convexo-concave tooth profile, in which circular arcs appear in the transverse section. Dyson et al. [13] presented mathematical model of Novikov internal and external gear mesh with one line of action. This model allowed researches to simulate gear tooth engagement with axis distance error. A mathematical model introduced by Litvin et al. [8] simulated gears with one line of action with axis distance as well as misalignment errors. In contrast to Dyson et al. [13] model, in which tooth profiles were circular arcs in transverse plane, Litvn et al. [8] assumed that tooth profiles were obtained by simulated fabrication by a rack-cutter with a parabolic cutting edge. Yang [14] presented a mathematical model of gear mesh with two lines of contact, in which tooth profiles were also fabricated by a rack-cutter. According to Radzevich [15], precise fabrication of this kind of gearing is impossible and they should be machined only by profiling i.e. by a disk-type formed milling cutter or a grinding wheel.

This paper focuses on research problems related to the measurement

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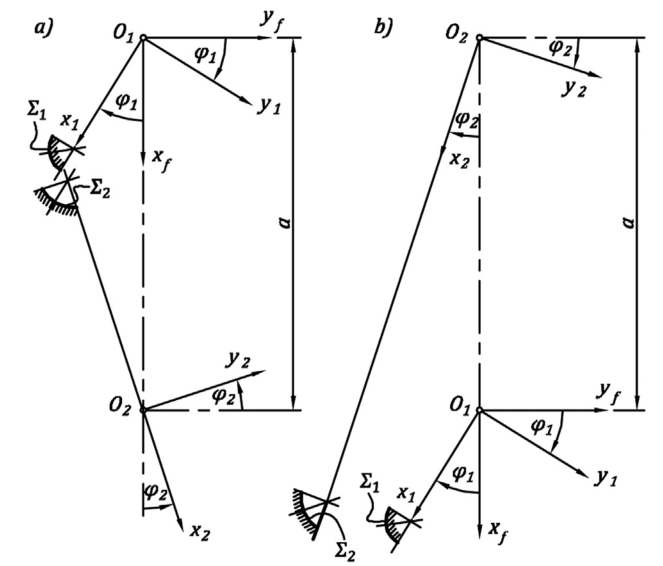

Fig. 1. Coordinate systems: (a) external gear mesh and (b) internal gear mesh.

and developing the universal mathematical model of external and internal Novikov gear mesh, which allows for the analysis of gears with axis distance and misalignment errors. In addition, it was assumed that tooth profiles are circular arcs in the transverse section. Part of this work related with mathematical modelling of this type of gear mesh is the expansion of previously developed model [2].

2. Mathematical model of gear mesh

The analysed gearbox is a parallel helical gear pair with internal and external gear mesh consisting of two Novikov gear wheels with convexo-concave tooth profiles. The pinion has a convex tooth profile and external gearing and is mated with a wheel, which has a concave tooth profile and external (Fig. 1a) or internal (Fig. 1b) gearing.

A stationary coordinate system x_f, y_f, z_f connected with a wheel case as well as coordinate systems connected with pinion and wheel x_1, y_1, z_1 and x_2, y_2, z_2 respectively were established. The pinion rotates clockwise around z_1-axis, which passes through point O_1 at the angle φ_1. The wheel rotates around z_2-axis, which passes through point O_2 at the angle φ_2, counter-clockwise in the case of external gearing (Fig. 1a) and clockwise in the case of internal gearing (Fig. 1b). The distance between centres a is also the distance between the centres of coordinate systems. The surface of pinion Σ_1 in coordinate system x_1, y_1, z_1 is represented by position vector $\bar{r}_1^{(1)}$.

Similarly, the surface of wheel Σ_2 in coordinate system x_2, y_2, z_2 is represented by position vector $\bar{r}_2^{(2)}$. Therefore, according to Fig. 1, surfaces of pinion and wheel teeth are represented in coordinate system x_f, y_f, z_f by Eqs. (1) and (2):

$$\bar{r}_1^{(f)} = M_{f1}\bar{r}_1^{(1)} \tag{1}$$

$$\bar{r}_2^{(f)} = M_{f2}\bar{r}_2^{(2)} \tag{2}$$

where M_{f1} is the homogeneous matrix of transformation from *1* to f, and M_{f2} is the homogeneous matrix of transformation from *2* to f.

If gear position errors resulting from assembly, manufacturing, deflection of shafts and bearings are to be taken into consideration, the coordinate system connected with a wheel must be translated through the x_f, y_f, z_f axes around Δa_x, Δa_y and Δa_z, and rotated around constant x_f, y_f axes respectively the angles κ_x and κ_y. For this purpose, auxiliary coordinate system x_h, y_h, z_h was established, as shown in Fig. 2.

Having taken gear position errors into account, the wheel will rotate around new translated and misaligned $z_h = z_2$ axis. According to Fig. 2, matrix of transformation is given by formula (3)

$$M_{f2} = M_{fh}M_{h2} \tag{3}$$

where M_{hf} is the homogeneous matrix of transformation from h to f, M_{h2} is the homogeneous matrix of transformation from *2* to h. Each matrix will be given as:

$$M_{f1} = \begin{bmatrix} \cos\varphi_1 & \sin\varphi_1 & 0 & 0 \\ -\sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$M_{h2} = \begin{bmatrix} \cos\varphi_2 & \mp\sin\varphi_2 & 0 & 0 \\ \pm\sin\varphi_2 & \cos\varphi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$M_{fh} = \begin{bmatrix} \cos\kappa_y & \sin\kappa_x\sin\kappa_y & \cos\kappa_x\sin\kappa_y & \pm a \pm \Delta a_x \\ 0 & \cos\kappa_y & -\sin\kappa_x & \Delta a_y \\ \sin\kappa_y & \sin\kappa_x\cos\kappa_y & \cos\kappa_x\cos\kappa_y & \Delta a_z \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{6}$$

The upper sign in all presented formulae refers to external gearing, while the lower one to internal gearing. Moreover, for surface

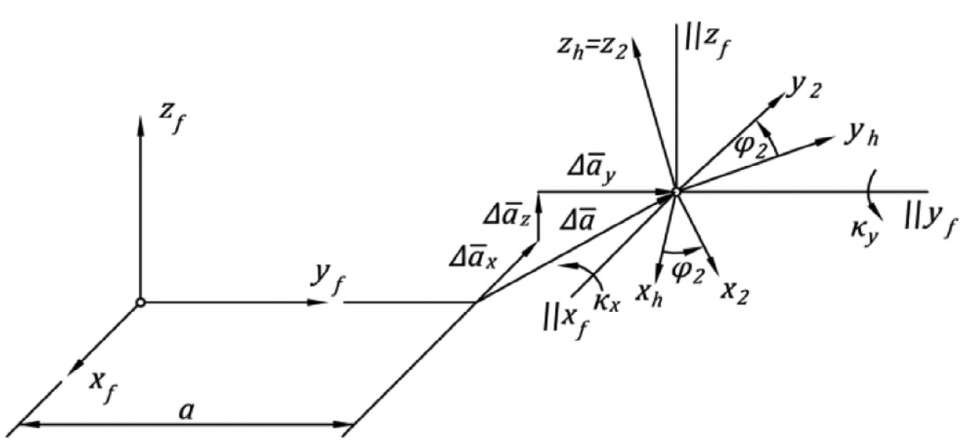

Fig. 2. Position and orientation of auxiliary coordinate system h (exampled on external gearing) [2].

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