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# Precision of evaluation methods in white light interferometry. Correlogram correlation method

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ABSTRACT

In this letter we suggest a method for the evaluation of a surface's topography which we call the correlogram correlation method. Employing a theoretical analysis as well as numerical simulations the method is proven to be the most accurate among available evaluation algorithms in the common case of uncorrelated noise. We obtain variance estimations for conventional methods and compare them to simulations. Experimental examples illustrate the superiority of the correlogram correlation method over the common envelope and phase methods.

## 1. Introduction

Since the beginnings of White Light Interferometry (WLI) it has been clear that WLI is a powerful tool to determine the topography of a surface [1]: The WLI signal is a correlogram wave packet *I*. The shift $z_0$ of its position on the scanning axis $z$ signals a change of the local height of the reflecting surface. There exist two established methods to localize $z_0$ on the axis $z$ [2]: the phase method originally introduced in the monochrome interferometry (PSI – Phase Shifting Interferometry) and the correlogram envelope evaluation method (CSI – Coherence Scanning Interferometry) which relies on the properties of the broad WLI signal spectrum. However, both methods harvest only parts of the information contained in a measured correlogram. As a consequence, the envelope evaluation methods suffer from low precision [2] and PSI is subjected to the $2\pi$ ambiguity of phase determination [3]. Numerous attempts exist to marry both procedures [4–6], but their success is limited [6], because the information contained in the correlogram is still only partly employed. To use the complete information one has to consider the full shape of the correlogram, which is achieved by the method presented here. We obtain and compare analytical and numerical estimations of noise variances of the common WLI-methods and the proposed method. Examples of practical application accomplish the study.

In the absence of noise/other disturbances – the only changes to the shape of the correlogram are contrast scaling and a shift of its position along the scanning axis. In order to locate the surface, one has to search for the expected correlogram pattern on the scanning axis. The best way to do so is to find the position of maximum correlation with a reference correlogram by calculating the cross-correlation function. The arising method, which we name correlogram correlation or, for the sake of brevity, CorCor method, is illustrated in Fig. 1. The idea to look for the position of a characteristic pattern in the measured correlogram has been in one way or another touched on by some authors, e.g. in [6–9], but the approach has been only considered as an additional procedure for specific purposes. The works [7,8] apply it to the special case of transparent film metrology. While [7,8] employ a complicated window shifting procedure combined with a least-square estimation to find the correlogram position, the authors of [6,9] do not consider the idea of a reference correlogram, but use a model correlogram instead, which results in loss of information and the deterioration of precision. In [6] correlations with a sequentially shifted model packet are used to determine the envelope position and the base harmonic phase distribution. On the whole, implementations of the principle of correlogram correlation are hitherto hardly to be found in WLI-applications, in spite of the fact that the correlation procedures are widely used in other areas. The reason is that in the area, it has never been pointed out that the method is just bound to be the most precise, which is easily shown mathematically.

An additional important advantage of the proposed method is the fact that it provides a direct criterion for the appropriateness of a correlogram measured on any point on the surface – the covariance with the reference correlogram at the best fitting position. This criterion is more informative and useful than the commonly employed criterion of the magnitude of maximum contrast [2]. The repeatability of surface height gauging is often used as the measure of WLI accuracy/uncertainty [6]. We use the noise-induced variance of height estimation as the direct measure of uncertainty both for numerical and experimental assessments.

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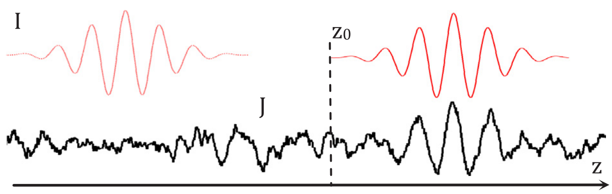


**Fig. 1.** The correlogram correlation (CorCor) method: search for the position $z_0$ of optimal fitting of a reference correlogram $I$ to a measured correlogram $J$. At $z_0$ the correlation between I and J is maximal.

## 2. Comparison of height estimation methods

### 2.1. Application of the maximum likelihood method to WLI signals

To see that the cross-correlation technique is the best way to ascertain the correlogram position let us obtain $z_0$ following the Maximum Likelihood Method (MLM) [10]. Let $I_j \equiv I(z_j - z_0)$ be the reference correlogram of the interferometer at measurement points $z_j$ shifted to the position $z_0$. Then for the measured correlogram $J_j$ holds

$$J_j = I_j(z_0) + \delta_j, \tag{1}$$

where $\delta_j$ represents a discrepancy between $J_j$ and the reference correlogram stemming from noise, which we assume to be uncorrelated and Gaussian. It is an appropriate assumption for many applications [11,12]: the camera shot noise is of this kind; the observed $N^{1/2}$ dependence of the height estimation variance on the number of measurement points N [2] indicates the absence of noise correlation. Following the MLM, among possible shift positions $z_0$ of $I_j$ we have to choose the one, at which the probability that the measured correlogram is constituted by the $I_j$ and noise $\delta_j$ is maximal. This probability is given by

$$\prod_{j=1}^{N} p_{\delta_j} = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^N \exp\left(-\sum_{j=1}^{N} \delta_j^2/2\sigma^2\right), \tag{2}$$

where $p_{\delta j}$ are the partial probabilities at the measurement points, and $\sigma^2$ is the noise dispersion which is supposed to be equal for all the points. The probabilities depend only on deviations, not on the signal values, because the correlograms are additive. Particularly, noise is added linearly. This fact substantiates the expression (2) and all the following derivations. The requirement of maximization of (2) results in the requirement of least-squares:

$$\sum \delta_j^2 \to \min, \quad \text{or} \quad \sum (J_j^2 - 2J_jI_j + I_j^2) \to \min \tag{3}$$

(here and below, if not specified otherwise the sums are over the set of measurement points). (3) is equivalent to the requirement of maximum covariance:

$$(J \otimes I)(z_0) \equiv \sum J_j I_j(z_0) \to \max_{z_0}. \tag{4}$$

Thus, the most probable shift of the correlogram and hence the most probable surface position is at a $z_0$, where the covariance of the measured and reference correlograms is maximal. In other words, to be in accordance with the MLM and to get the surface position one has to calculate the cross-correlation function and to find the position of its maximum. This is the correlogram correlation (CorCor) method. Let us emphasize: according to the above derivation no other procedure can give a more accurate estimation of surface height in the sense of its probability. Hence, the robustness to noise of this estimation procedure cannot be surpassed [10]. Note that this statement is correct for large noise amplitudes as well as for low.

The fact that the correlogram values are only known on the grid of measurement points $z_j$, which in WLI is typically rare, does not mean that the maximum in (4) can be found only at one of these points. It means instead that frequencies above the Nyquist limit are absent in correlograms. This limitation does not prevent us from precisely finding the cross-correlation maximum. In this study we have calculated the cross-correlation function (4) on the discrete grid and then interpolated it using an interpolation method which preserves the spectrum of the interpolated function. The details and substantiations necessary to implement the correlogram correlation method are given in [11].

### 2.2. Height evaluation methods and their variances

The variance of the CorCor method is obtained as follows: Suppose a sample $J_j$ is the result of noise added to a pattern $I_j$, the pattern being shifted to a position $z_0$. The probability $p$ of the occurrence of this sample depends on the value of $z_0$. Then, the Cramer-Rao bound [10] gives an estimator of the $z_0$ variance ($E$ is the operation of taking the statistical expectation value):

$$\mathrm{var}(z_0) \leqslant -1/E(\partial^2 \ln p/\partial z_0^2). \tag{5}$$

Using (1) and the probabilities given by (2), we obtain:

$$\ln Cp = -\sum \frac{\delta_j^2}{2\sigma^2}; \frac{\partial^2 \ln p}{\partial z_0^2} = \frac{\partial}{\partial z_0}\left[-\frac{1}{2\sigma^2} - \sum 2\delta_j \frac{\partial I_j(z_0)}{\partial z_0}\right]$$
$$= \frac{1}{\sigma^2}\sum\left\{-\left[\frac{\partial I_j(z_0)}{\partial z_0}\right]^2 + \delta_j \frac{\partial^2 I_j(z_0)}{\partial z_0^2}\right\}.$$

C is a constant. Because of $E(\delta_j) = 0$, the variance is:

$$\mathrm{var}(z_0) = \sigma^2/\sum\left[\frac{\partial I_j(z_0)}{\partial z_0}\right]^2 = \sigma^2/\sum\left[\frac{dI}{dz}\bigg|_j\right]^2. \tag{6}$$

In the CSI, a reliable part of the envelope called "half-height envelope" is commonly found above the half-level of its maximum deviation. There are two established methods to estimate the envelope position, the parabola fitting and the centroid method. In the former a parabola is least-square fitted to the half-height envelope and its maximum position is taken as $z_0$, in the latter $z_0$ is equal to the half-height envelope's center of gravity. A lower bound for the height estimation of the parabola fitting can be found similarly to (6). For the centroid method it can be obtained directly. The two expressions are [11]:

$$(a)\mathrm{var}(z_0) = \sigma^2/\sum[(d\widehat{E}_0/dz)|_j]^2; (b)\mathrm{var}(z_0) = \sigma^2\sum(j - z_0)^2/\left(\sum \widehat{E}_{0j}\right)^2, \tag{7}$$

where $\widehat{E}_0$ and $\widehat{E}$ are the envelopes of reference ($I$) and measured ($J$) correlograms. The sum is over the half-height envelope. Here and below we take the discretization step $\Delta z$ as a unit of z, so $z_j = j$.

The PSI basically translates the phase φ of a harmonic with wavelength λ to height according the obvious relation $z = (1/2\pi)\lambda\varphi$. For the k^th digital harmonic it is $z = (1/2\pi)(N/k)\varphi_k$. The application of the MLM to a set of harmonics $k_1 \leq k \leq k_2$ [12] gives the height estimation

$$z_0 = -\frac{\sum_{k=k_1}^{k_2} |X_k|^2(2\pi k/N)\phi_k}{\sum_{k=k_1}^{k_2} |X_k|^2(2\pi k/N)^2}, \tag{8}$$

where $X_k$ are the complex amplitudes of harmonics. Their norms are used as weights while fitting the phases with a straight line. Different choices of $k_1$ and k2 are possible and suite different situations. Below, in our simulations, we consider the following variants: $k_1 = 1$, $k_2 = N/2$ (complete spectrum); $k_1 = k_2 = k_0$, where $k_0$ is the index of the harmonic with the maximum amplitude (main harmonic); and $k_{1,2}$ determined as indices of the first and last harmonics with amplitudes not less than 5% of the maximum amplitude (main spectrum). Besides, we consider a fitting without weighting over the main spectrum setting $|X_k| = 1$ in (8) and the fitting of phase gradients instead of the phases themselves [6], where again no weights are used and only the main spectrum employed.The variance bound corresponding to (8) is found in [12,13], and can be written as