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Tension determination for suspenders of arch bridge based on multiple vibration measurements concentrated at one end

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ABSTRACT

Considering the specific arrangement of boundary constraints for the suspenders of arch bridge, this research extends a recently developed method to carry out the determination of suspender tension simply with multiple measurements concentrated at one end. The virtual hinged boundary of the equivalent cable is first decided by the optimal shifting parameter solved from best fitting the identified mode shape ratios with an unsymmetrical formulation. The effective vibration length for the suspender with a hinged connection at the higher end is taken as the distance between the virtual and actual hinged boundaries to directly estimate its tension. On the other hand, an iteration algorithm using a finite element model is developed to find the convergent tension for the suspender with a fixed constraint at the arch end. The feasibility and accuracy of the proposed method are extensively verified with demonstrative numerical examples, laboratory experiments, and practical applications in real bridges.

1. Introduction

The tension variations in stay cables of cable-supported bridges, external tendons of box girder bridges, or suspenders of through-type arch bridge play an important role in the structural health monitoring of the corresponding bridges. A variety of devices have been applied in engineering practice to directly determine the tension of these critical force-transmitting members, but several types of difficulties are also confronted. Hydraulic jacks are typically used during the stressing stage of construction and load cells can be installed at the same stage for long-term monitoring. However, the accuracy of the former is usually questioned and a considerably high cost has to be paid for the latter. Much cheaper strain gauges can be attached at the anchorage or on the cable strand to roughly estimate the tension if durability is not of great concern. With a price of complicated fabrication, fiber Bragg grating (FBG) sensors have been embedded inside the cable cross-section to detect the variation of tension by measuring the associated strain [1,2]. Taking advantage of the sensitivity in the magnetic permeability of steel material to its stress variation, elasto-magnetic (EM) sensors were recently developed to conduct the tension measurement of stay cables [3–5] with the requirement of deliberate calibrations in the laboratory or field.

Blessed by the one-dimensional geometry, the analysis and measurement for a stay cable, external tendon, or suspender is relatively simple compared to the other types of civil structures. Exploiting this feature together with the benefit of easy operation, the ambient vibration method is more popularly adopted for the determination of tension in practical applications than the direct approaches previously mentioned in either the construction [6,7] or service stage [8–10]. Such an indirect approach regularly starts from identifying the cable frequencies from the ambient vibration measurements with traditional contact or advanced non-contact sensors [11–15]. With these identified frequencies as well as the given length and mass per unit length, the tension can be conventionally estimated from an analytical formula based on the string theory where the simplified model of a transversely vibrating string with hinged boundary conditions is assumed. More intricate analytical or empirical formulas have been developed to enhance the accuracy of this method by further considering the influences of flexural rigidity [16–19], gravity sag [20–22], and complicated boundary conditions [23–28]. Or alternatively, finite element (FE) analysis has also been employed in a few numerical approaches for deciding the optimal values of tension, flexural rigidity, and other parameters such that all the identified modal frequencies can be best fitted [29–32].

Other than dealing with the above modeling issues, the appropriate selection of vibration length and flexural rigidity that can accurately represent the real vibration behavior is probably more important for an accurate estimation of tension with a given formula. In engineering

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practice, rubber constraints and special anchorage systems are commonly installed at both ends of stay cables. Moreover, intermediate diaphragms are frequently associated with the application of external tendons to alter the direction of tendon for an optimal prestressing arrangement inside the box girder. These designs certainly complicate the boundary conditions and induce a great difficulty in the selection of effective vibration length, which is normally the most sensitive parameter to determine the tension.

Motivated by tackling the above problem, a novel idea of combining the mode shape ratios with the modal frequencies identified from ambient vibration measurements was recently explored by the authors to develop an expedient and accurate method for tension determination [33–36]. Multiple synchronized signals were first processed to obtain the mode shape ratios at various sensor locations for each observable mode. These ratios were subsequently utilized to fit the sinusoidal mode shapes and independently decide the effective vibration length from optimization procedures. Other researchers have also begun paying attention to this new concept of effective vibration length in the past few years [37–41]. According to the obtained vibration lengths for several selected modes and the corresponding identified frequencies, the tension and flexural rigidity can then be solved by simple linear regression techniques. While a crucial restriction of symmetric boundary constraints at both ends was initially imposed in the optimization of effective vibration length for simplicity [33,34], this method was further generalized by introducing an extra shifting parameter in the shape function to effectively consider the unsymmetrical boundary constraints [35,36]. Such an unsymmetrical formulation, however, would generate a complex objective function full of local minima and difficult to obtain a reasonable optimal value for the effective vibration length if all the sensors are installed near one end. It was also discovered that this obstacle can be overcome with the addition of one sensor near the other end. Even though such a recipe may not cause particular troubles for the case of external tendon, it is definitely not convenient for a stay cable or a suspender of arch bridge where the help of a hired crane is usually necessary for the sensor installation close to the far end.

In general, the suspenders of through-type arch bridge are subjected to smaller fatigue stress and their anchorage systems are thus not as complicated as those for the stay cables. Taking the arch bridges in Taiwan majorly constructed for scenic reasons as examples, the length of their suspenders is typically less than 30 m. For these short cables, a protection tube with a steel or rubber cap on the top is regularly adopted at the deck end to seal the anchorage for preventing water penetration. At the other end connected to the arch, the hinged or fixed connection is frequently utilized for simple anchorage [42]. A number of works have been conducted to determine the tension for suspenders of arch bridge with empirical formulas [42,43], finite element analysis [44], or advanced numerical techniques such as neural networks [45,46]. Considering the specific arrangement of boundary constraints at the both ends of a suspender, the current study is aimed to extend the recently developed methodology [33–36] for further improving the accuracy and convenience of its tension estimation merely with multiple measurements concentrated at one end. More theoretical background for excluding the effects of uncertain boundary constraints based on an equivalent cable with hinged constraints at both ends is first elaborated in this paper. Several numerical issues in optimization process will then be discussed to provide the basis for establishing an algorithm to combine with finite element analysis. Finally, the numerical verification, experimental validation, and practical applications for this new method will be presented in order.

2. Methodology based on multiple vibration measurements

The cable vibration equation and its solutions are first examined in this section, followed by probing the theoretical backgrounds for the methodology based on multiple vibration measurements to estimate the tension [33–36]. In addition, the formulation for such an approach based on multiple ambient vibration measurements is briefly reviewed.

2.1. Cable vibration equation and its solutions

Since the suspenders of an arch bridge are typically shorter than the stay cables and installed in the vertical direction, the associated effect of gravity sag is undoubtedly negligible. Without the consideration of sag-extensibility, the transverse displacement $v(x, t)$ of a cable subjected to an axial tension T and under free vibration is a function of axial coordinate x and time t to be governed by the following equation of motion:

$$EI\frac{\partial^4 v}{\partial x^4} - T\frac{\partial^2 v}{\partial x^2} + \overline{m}\frac{\partial^2 v}{\partial t^2} = 0 \tag{1}$$

where $\overline{m}$ is the mass per unit length, E denotes the Young's modulus, and I represents the cross-sectional area moment of inertia. For solving Eq. (1), separation of variables is conveniently applied to assume $v(x,t) = \phi(x)V(t)$ and then yield

$$\begin{cases} \ddot{V}(t) + \omega^2 V(t) = 0 \\ \phi^{iv}(x) - \varepsilon\phi''(x) - \beta^2\omega^2\phi(x) = 0 \end{cases} \tag{2}$$

where $\varepsilon \equiv T/EI$ and $\beta^2 \equiv \overline{m}/EI$. Eventually, $V(t)$ and $\phi(x)$ can be determined [16] to be:

$$V(t) = A_1\sin\omega t + A_2\cos\omega t \tag{3}$$

and

$$\phi(x) = C_1\sin\delta x + C_2\cos\delta x + C_3\sinh\gamma x + C_4\cosh\gamma x \tag{4}$$

where

$$\delta = \sqrt{\sqrt{\beta^2\omega^2 + \frac{\varepsilon^2}{4}} - \frac{\varepsilon}{2}} \text{ and } \gamma = \sqrt{\sqrt{\beta^2\omega^2 + \frac{\varepsilon^2}{4}} + \frac{\varepsilon}{2}} \tag{5}$$

For the simple case of the cable with hinged boundary constraints at both ends, it is necessary to satisfy the conditions

$$\begin{cases} \phi(0) = C_2 + C_4 = 0 \\ \phi''(0) = -\delta^2 C_2 + \gamma^2 C_4 = 0 \\ \phi(L) = C_1\sin\delta L + C_2\cos\delta L + C_3\sinh\gamma L + C_4\cosh\gamma L = 0 \\ \phi''(L) = -\delta^2 C_1\sin\delta L - \delta^2 C_2\cos\delta L + \gamma^2 C_3\sinh\gamma L + \gamma^2 C_4\cosh\gamma L = 0 \end{cases} \tag{6}$$

where L signifies the cable length. The simultaneous equations in Eq. (6) can be easily solved as $C_2 = C_3 = C_4 = 0$ and

$$\sin\delta L = 0 \tag{7}$$

Each of the infinitely many solutions in Eq. (7) is specified by

$$\delta_k = \frac{k\pi}{L}, \quad k = 1,2,3,\cdots \tag{8}$$

and further substituted into the former part of Eq. (5) to result in an analytical formula for the modal frequencies of cable:

$$\left(\frac{f_k}{k}\right)^2 = \left(\frac{\omega_k}{2k\pi}\right)^2 = \frac{T + \frac{k^2\pi^2 EI}{L^2}}{4\overline{m}L^2}, \quad k = 1,2,3,\cdots \tag{9}$$

where $f_k = \omega_k/2\pi$ denotes the natural frequency of the k-th mode in Hz. Besides, the mode shape corresponding to each modal frequency f_k is solely in the form of sine functions:

$$\sin\frac{k\pi x}{L}, \quad k = 1,2,3,\cdots \tag{10}$$

For the more complicated case of the cable with fixed boundary constraints at both ends, the corresponding boundary conditions become