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Comparison of differentiation schemes for the velocity and acceleration estimations of a pneumatic system

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Abstract: In this paper, various type of differentiators are studied. In the context of velocity and acceleration estimations of a pneumatic system, a comparison is made between numerical methods, based on classical or algebraic approaches, and a high order sliding mode differentiator.

Keywords: Differentiation, high order sliding mode, pneumatic actuator.

1. INTRODUCTION

These last decades, numerous results have been proposed for the control of electropneumatic systems, the main part of these results being based on state feedback approaches: input-output linearization based control (Brun et al. (1999)), sliding mode control (Girin et al. (2009); Laghrouche et al. (2006); Plestan et al. (2013); Shtessel et al. (2012); Smaoui et al. (2006 b); Taleb et al. (2013)), backstepping control (Smaoui et al. (2006 a,b)), etc. However, these controllers require the measurement of state variables. In practice, some state variables are not easy to be measured directly by the sensors, like the velocity and the acceleration, whereas they are required to compute the control law. In order to overcome this difficulty and also with the objective of minimizing the number of sensors, some states estimation schemes can be proposed (Yan et al. (2014)). One solution is the use of nonlinear state observer. In Bornard et al. (1991), a high gain observer is proposed for a class of nonlinear system. A sliding mode observer is used by Pandian et al. (2002), so as to estimate the chamber pressure for pneumatic actuators.

An alternative method to estimate the system state is the numerical differentiation. According to the studies in Gauthier and Kupka (1994), for an observable nonlinear system, any state variables is a function of finite number of time derivatives of the control and output variables. Furthermore, the use of numerical differentiation schemes enables a model-independent derivation. In recent years, numerous technics have been proposed for the problem of numerical differentiation. A robust exact differentiator based on 2-sliding algorithm (see Levant (1993)) is proposed by Levant (1998). It allows to estimate the first order derivative of a bounded noisy signal. Such a differentiator is used by Smaoui et al. (2005) for the acceleration estimation of a pneumatic system. The sampling feature of the differentiation computation is also taken into account by Plestan and Glumineau (2010). The sliding mode differentiator is generalized to the higher-order sliding mode differentiator (Levant (2003)), which allows to estimate the k-th order derivative of a bounded noisy signal. Moreover, the so called *chattering* phenomenon is reduced through the high order sliding mode theory. Another kind of differentiator based on algebraic parametric estimation technics is proposed by Mboup et al. (2007): a truncated Taylor expansion and calculations in operational domain are used to obtain the approximations of the finite order derivatives of a noisy signal. In Liu et al. (2009), the error analysis for such a differentiator is done.

The objective of the paper is to apply some differentiation approaches for the velocity and acceleration estimations of a pneumatic system. Four methods are considered here. Two of them are based on classical approaches: one is based on the backward-difference formula, the other is based on a three-point formula. The third differentiator is developed from results of Mboup et al. (2007) by using an algebraic approach. Finally, the higher-order sliding mode differentiator proposed by Levant (2003) is considered. The paper is organized as follows. In the second section, numerical differentiation methods are presented. The differentiator based on high order sliding mode is exposed in the third section. In the fourth one, the experimental set-up of the pneumatic system is described. Furthermore, experimental results of velocity and acceleration estimations are presented and a comparison between the four approaches is made with different working conditions.

2. NUMERICAL DIFFERENTIATION

2.1 Classical approach

The principle of the classical numerical differentiation method presented in this section is to estimate the time derivative of a function $f(t)$ by calculating the derivative of an interpolating polynomial that fits $f(t)$ over an interval I.

The next theorem establishes a formula to approximate $f'(t_0)$ from the sampling points at instants $t_{-j},\dots,t_0,\dots,t_k$ (denote $I=[t_{-j},t_k]$).

Theorem 1. (Burden and Faires (2011)) Let $n=j+k$ and $f(t)\in\mathcal{C}^{n+1}(I)$. Then, the first order derivative at $t=t_0$, $f'(t_0)$ is given by the following $(n+1)$-point formula

$$f'(t_0)=\sum_{i=-j}^{k} f(t_i)\mathcal{L}'_{n,i}(t_0)+\frac{f^{(n+1)}(\xi)}{(n+1)!}\prod_{i=-j,i\neq 0}^{k}(t_0-t_i)\ , \quad (1)$$

where $\xi\in I$ is an unknown instant and the Lagrange polynomial $\mathcal{L}_{n,i}$ associated with $t_{-j},\dots,t_k$, is defined, for $i=-j,\dots,k$ by

$$\mathcal{L}_{n,i}(t)=\prod_{l=-j,l\neq i}^{k}\frac{t-t_l}{t_i-t_l}\ . \quad (2)$$

■

Proof. By the Lagrange interpolation theorem, for all $t\in I$, there exists $\xi\in I$ such that the following equation holds

$$f(t)=\sum_{i=-j}^{k} f(t_i)\mathcal{L}_{n,i}(t)+\frac{(t-t_{-j})\dots(t-t_k)}{(n+1)!}f^{(n+1)}(\xi)\ . \quad (3)$$

Then, the $(n+1)$-point differentiation formula can be obtained by differentiating both sides of (3) at $t=t_0$. ■

Remark 1.

- With two sampling points, *i.e.* $j=-1$ and $k=0$, one obtains the classical backward-difference formula
$$f'(t_0)=\frac{f(t_0)-f(t_0-h)}{h}+\frac{h}{2}f''(\xi) \quad (4)$$
where $h=t_0-t_{-1}$ represents the sampling period. The term $\frac{h}{2}f''(\xi)$ which is proportional to the sampling period h, gives the approximation accuracy.
- In order to improve the accuracy, more sampling points are considered. Assume now that $j=-2, k=0$ and the sampling period is uniform (*i.e.* $t_0-t_{-1}=t_{-1}-t_{-2}=h$). Then, one gets the three-point formula
$$f'(t_0)=\frac{3f(t_0)-4f(t_0-h)+f(t_0-2h)}{2h}+\frac{h^2}{3}f^{(3)}(\xi)\ . \quad (5)$$
In this expression, the accuracy is proportional to h^2. Since the sampling period h is generally smaller than 1s, one gets a better accuracy.

Note that the smaller h is, the better accuracy one gets. However, in the case of a noisy signal $f(t)$, the differentiation becomes more sensitive to this noise. From a practical point of view, in controlled systems, the sampling period should be tuned to satisfy the control law. So as to improve these differentiators, one adds a parameter $H=nh$ where n is a positive integer. From expressions (4)-(5), one thus deduces two formulations for the estimate of the first derivative $\hat{f}'(t)$ of a signal f at current time t.

$$\hat{f}'(t)=\frac{f(t)-f(t-H)}{H} \quad (6)$$

and

$$\hat{f}'(t)=\frac{3f(t)-4f(t-H)+f(t-2H)}{2H}\ . \quad (7)$$

Expressions (6)-(7) will be experimentally tested on the pneumatic system (Section 4), in order to estimate the velocity and the acceleration.

2.2 Algebraic approach

An alternative approach to estimate the derivatives of a (possibly noisy) signal is proposed by Mboup et al. (2007). It is based on Taylor expansion and Laplace transform and provides the advantage of simultaneously getting estimates of the higher order derivatives of the signal. Next theorem presents this differentiator.

Theorem 2. (Mboup et al. (2007)) Let N be a positive integer. Assume that $y(t)=f(t)+n(t)$ is a noisy signal defined on $[0,+\infty]$, which consists of a basic signal $f(t)$ and a noise $n(t)$.

Then, the estimates of the i-th order time derivatives $\hat{f}^{(i)}(0)$, $i=0,\dots,N$ of $f(t)$ at $t=0$ are given by the following general expression

$$P(T)\begin{bmatrix}\hat{f}(0)\\ \hat{f}'(0)\\ \vdots\\ \hat{f}^{(N)}(0)\end{bmatrix}=\int_0^T Q(\tau)y(\tau)d\tau \quad (8)$$

where T is the size of the estimation window. The nonzero elements of the triangular matrix $P(T)$ are given, for $i=0,\dots,N$, $j=0,\dots,N-i$, by

$$P(T)_{i,j}=\frac{(N-j)!}{(N-i-j)!}\frac{T^{i+j+1}}{(i+j+1)!}\ , \quad (9)$$

and the elements of the integral term are

$$Q(\tau)_i=\sum_{l=0}^{i}\binom{i}{l}\binom{N+1}{l}(T-\tau)^l(-\tau)^{i-l} \quad (10)$$

with $i=0,\dots,N$. ■

Remark 2.

- A more general result is proved in Mboup et al. (2007) with an additional parameter ν . For simplicity, we assume here $\nu=N+2$ and we retain this choice throughout the remainder of the paper.
- The expansion (8) is not a causal differentiator: it requires the signal values $y(t)$ for $t>0$ in order to reconstruct the derivatives at $t=0$. It means that the future signal values should be known to estimate the derivative in the current time.

So as to estimate the derivatives at the current time t from $y(\tau),\tau<t$, one adapts the differentiator (8) by the following way.

Corollary 1. Consider the same assumptions as in Theorem 2. The estimates $\hat{f}^{(i)}(t)$ for $i=0,\dots,N$, are given by the expression

$$\widetilde{P}(T)\begin{bmatrix}\hat{f}(t)\\ \hat{f}'(t)\\ \vdots\\ \hat{f}^{(N)}(t)\end{bmatrix}=\int_0^T Q(\tau)y(t-\tau)d\tau \quad (11)$$

with

$$\widetilde{P}(T)_{i,j}=(-1)^jP(T)_{i,j} \quad (12)$$

and $P(T)_{i,j}$, $Q(\tau)_i$ defined by (9) and (10). ■