Proceedings of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Maximum likelihood calibration of a magnetometer using inertial sensors

Manon Kok * **Thomas B. Schön** **

* *Division of Automatic Control, Linköping University, SE-581 83 Linköping, Sweden (email: manko@isy.liu.se)*
** *Department of Information Technology, Uppsala University, SE-751 05 Uppsala, Sweden (e-mail: thomas.schon@it.uu.se)*

Abstract: Magnetometers and inertial sensors (accelerometers and gyroscopes) are widely used to estimate 3D orientation. For the orientation estimates to be accurate, the sensor axes need to be aligned and the magnetometer needs to be calibrated for sensor errors and for the presence of magnetic disturbances. In this work we use a grey-box system identification approach to compute maximum likelihood estimates of the calibration parameters. An experiment where the magnetometer data is highly disturbed shows that the algorithm works well on real data, providing good calibration results and improved heading estimates. We also provide an identifiability analysis to understand how much rotation is needed to be able to solve the calibration problem.

Keywords: Magnetometers, calibration, inertial sensors, maximum likelihood, grey-box system identification, sensor fusion.

1. INTRODUCTION

Inertial sensors (3D accelerometers and 3D gyroscopes) in combination with 3D magnetometers are in many applications used to obtain orientation estimates using an extended Kalman filter (EKF). When the sensor is subject to low accelerations, the accelerometer measurements are dominated by the gravity component from which it is possible to deduce information about the inclination. The magnetometer measures the local magnetic field vector and its horizontal component can be used to obtain heading information. The orientation estimates are only accurate when the sensor axes of the inertial sensors and the magnetometers are aligned and when the magnetometer is properly calibrated. This calibration consists of two parts. First, the magnetometer needs to be calibrated for errors inherent in the sensor, for instance non-orthogonality of the magnetometer sensor axes. Second, it needs to be calibrated for the presence of magnetic disturbances. These magnetic disturbances are frequently present due to the mounting of a magnetometer and cause a constant disturbance that needs to be calibrated for. When using a magnetometer it is therefore always advisable to calibrate it before use.

Our main contribution is a new practical algorithm for calibration of a magnetometer when we also have access to measurements from inertial sensors rigidly attached to the magnetometer. The algorithm implements a maximum likelihood (ML) estimator to find the calibration parameters. These parameters account for magnetometer sensors errors, the presence of constant magnetic disturbances caused by mounting the magnetometer close to magnetic objects and misalignments between the magnetometer and the inertial sensors. An illustration of the results obtained from calibrating a magnetometer using the proposed algorithm is available in Fig. 1.

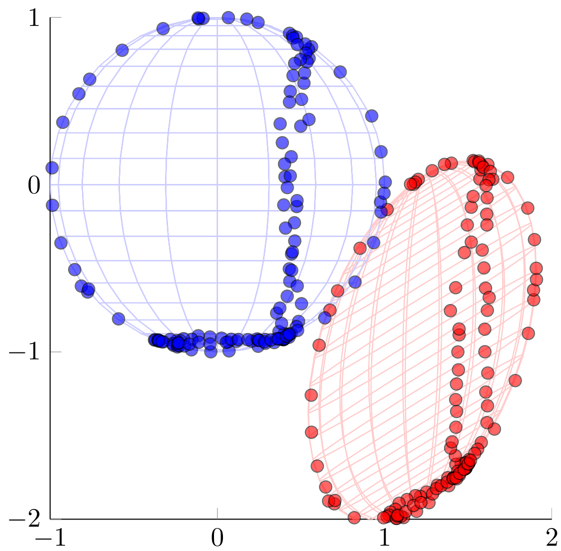

Fig. 1. Calibration results for experimental data where the original magnetometer data is plotted in red and the calibrated magnetometer data is plotted in blue.

In many practical applications, mounting the magnetometer onto for instance a car or a boat severely limits the rotational freedom of the sensor. A secondary contribution of this work is a quantification of how much rotation is needed to be able to solve the calibration problem. This is done via an identifiability analysis, deriving how much rotation is needed in the case of perfect measurements.

⋆ This work is supported by MC Impulse, a European Commission, FP7 research project and CADICS, a Linnaeus Center funded by the Swedish Research Council (VR).

Many recent magnetometer calibration approaches are based on ellipse fitting. These calibration algorithms use the fact that when rotating a magnetometer, its measurements should lie on a sphere if the magnetometer measures a constant local magnetic field vector, i.e. the norm of the magnetic field should be constant. The presence of magnetic disturbances or magnetometer sensor errors leads to an ellipsoid of data instead. Many calibration algorithms are used to obtain improved orientation estimates. Hence, the radius of the sphere, i.e. the actual magnitude of the local magnetic field vector, is irrelevant and we can without loss of generality assume that its norm is equal to 1. The calibration has therefore been successful when the calibrated measurements can be seen to lie on a (unit) sphere (recall Fig. 1). The problem of fitting an ellipsoid of data to a sphere was considered for example by Gander et al. [1994]. As shown by Markovsky et al. [2004] the ordinary least squares estimate is inconsistent when using measurements corrupted by noise. Different calibration approaches have been developed to overcome this problem, see e.g. Gebre-Egziabher et al. [2006], Renaudin et al. [2010].

An ellipse fitting method can only solve the calibration up to an unknown rotation. Combining the magnetometer with inertial sensors requires the sensor axes to be aligned, i.e. the sphere needs to be oriented such that the magnetometer sensor axes are aligned with the inertial sensor axes. A few recent approaches add a second step to the calibration procedure to estimate the misalignment between the magnetometer and the inertial sensor axes [Vasconcelos et al., 2011, Li and Li, 2012, Salehi et al., 2012, Bonnet et al., 2009]. They first use an ellipse fitting approach and determine the misalignment between the magnetometer and the inertial sensor axes in a second step. These approaches generally use the accelerometer measurements for estimating the misalignment, but discard the gyroscope measurements. Troni and Whitcomb [2013] focus on using the gyroscope measurements to obtain an estimate of the misalignment. Our work follows a similar approach, but makes use of both the gyroscope and the accelerometer measurements and aims at obtaining a maximum likelihood estimate by combining both steps into a nonlinear optimization problem.

2. PROBLEM FORMULATION

Our solution to the magnetometer calibration problem makes use of a nonlinear state space model describing the sensor's orientation q_t and its measurements,

$$q_{t+1} = f_t(q_t, u_t, \theta) + B(q_t)v_t(\theta), \tag{1a}$$

$$y_t = h_t(q_t, \theta) + e_t(\theta). \tag{1b}$$

Here, q_t denotes the state variable representing the sensor orientation encoded using a unit quaternion. Furthermore, u_t and y_t denote observed input and output variables, respectively. The dynamics is denoted by $f_t(\cdot)$ and the measurement model is denoted by $h_t(\cdot)$. Finally, v_t and e_t represent mutually independent i.i.d. process and measurement noise, respectively and $B(q_t)$ is a matrix describing how the noise v_t affects the state q_t. The model is introduced in detail in Section 3.

The calibration problem is formulated as a maximum likelihood grey-box system identification problem. Hence, based on N measurements of the observed input $u_{1:N} = \{u_1, \dots, u_N\}$ and output $y_{1:N} = \{y_1, \dots, y_N\}$ variables, find the parameters θ that maximize the likelihood function,

$$\widehat{\theta}^{\mathrm{ML}} = \arg\max_{\theta \in \Theta} \, p_\theta(y_{1:N}), \tag{2}$$

where $\Theta \subseteq \mathbb{R}^{n_\theta}$. Using conditional probabilities and the fact that the logarithm is a monotonic function we have the following equivalent formulation of (2),

$$\widehat{\theta}^{\mathrm{ML}} = \arg\min_{\theta \in \Theta} \, -\sum_{t=1}^{N} \log p_\theta(y_t \mid y_{1:t-1}), \tag{3}$$

where we have used the convention that $y_{1:0} \triangleq \emptyset$. The ML estimator (3) enjoys well-understood theoretical properties including strong consistency, asymptotic normality, and asymptotic efficiency [Ljung, 1999]. The state space model (1) is nonlinear, implying that there is no closed form solution available for the one step ahead predictor $p_\theta(y_t \mid y_{1:t-1})$. This can be handled using sequential Monte Carlo methods (e.g. particle filters and particle smoothers), see e.g. Schön et al. [2011], Lindsten and Schön [2013]. However, for the magnetometer calibration problem under study it is sufficient to make use of a more pragmatic approach; we simply approximate the one step ahead predictor using an extended Kalman filter (EKF). The result is

$$p_\theta(y_t \mid y_{1:t-1}) \approx \mathcal{N}\left(y_t \mid \widehat{y}_{t|t-1}(\theta), S_t(\theta)\right), \tag{4}$$

where $\mathcal{N}\left(y_t \mid \widehat{y}_{t|t-1}(\theta), S_t(\theta)\right)$ denotes the probability density function for the Gaussian random variable y_t with mean value $\widehat{y}_{t|t-1}(\theta)$ and covariance $S_t(\theta)$. Here, $S_t(\theta)$ is the residual covariance from the EKF [Gustafsson, 2012]. Inserting (4) into (3) results in the following optimization problem,

$$\min_{\theta \in \Theta} \frac{1}{2} \sum_{t=1}^{N} \|y_t - \widehat{y}_{t|t-1}(\theta)\|^2_{S_t^{-1}(\theta)} + \log\det S_t(\theta), \tag{5}$$

which we can solve for the unknown parameters θ. The problem (5) is non-convex, implying that a good initial value for θ is required.

3. MODELS

3.1 Dynamic model

We model the orientation q_t as the sensor's orientation from the body frame b to the navigation frame n at time t, expressed as q_t^{nb}. The body frame b is the coordinate frame of the inertial sensor with its origin in the center of the accelerometer triad and its axes aligned with the inertial sensor axes. The navigation frame n is aligned with the earth gravity and the local magnetic field vector. The dynamic model of the orientation (1a) takes the gyroscope measurements as an input, which are modeled as

$$y_{\omega,t} = \omega_t + \delta_\omega + v_{\omega,t}, \tag{6}$$

where ω_t denotes the angular velocity, δ_ω denotes the gyroscope bias and $v_{\omega,t} \sim \mathcal{N}(0, \Sigma_\omega)$. The dynamic equation for the orientation states is then given by [Gustafsson, 2012]

$$q_{t+1}^{\mathrm{nb}} = \left(\mathcal{I}_4 + \tfrac{T}{2} L(y_{\omega,t} - \delta_\omega)\right) q_t^{\mathrm{nb}} + \tfrac{T}{2} \bar{L}(q_t^{\mathrm{nb}}) v_{\omega,t}. \tag{7}$$

Here, $\mathcal{I}_4$ denotes a 4×4 identity matrix, T denotes the sampling time, the matrices $\bar{L}(q_t^{\mathrm{nb}})$ and $L(y_{\omega,t} - \delta_\omega)$ are given by