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Transmission Power Scheduling for Energy Harvesting Sensor in Remote State Estimation

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Abstract: We study remote estimation in a wireless sensor network. Instead of using a conventional battery-powered sensor, a sensor equipped with an energy harvester which can obtain energy from the external environment is utilized. We formulate this problem into an infinite time-horizon Markov decision process and provide the optimal sensor transmission power control strategy. In addition, a sub-optimal strategy which is easier to implement and requires less computation is presented. A numerical example is provided to illustrate the implementation of the sub-optimal policy and evaluation of its estimation performance.

1. INTRODUCTION

Wireless sensors network (WSN) has been a hot research topic in recent years. Both theoretical results and practical applications are growing rapidly. Compared with traditional wired sensors, wireless sensors provide many advantages such as low cost, easy installation, and self-power. In a WSN, sensors are typically equipped with batteries and expected to work for a long time (Yick et al. [2008]). Thus, the energy constraint is an inevitable issue. In some applications, the amounts of sensors can be quite large (e.g., environment monitoring) or sensors may be located in dangerous environments (Ho and Zhang [2012]) (e.g., chemical industry), making the replacement of batteries difficult or even impossible.

To deal with energy aspects of WSN, one possible way is to develop more efficient sensor energy power control methods to make the best use of the batteries (Aziz et al. [2013], Pantazis and Vergados [2007]). An alternative way is to replace the conventional battery-powered sensor with sensors equipped with an energy harvester. The technology of energy harvesting refers to obtaining energy from the external environment or other types of energy sources (e.g., body heat, solar energy, piezoelectric energy, wind energy) and converting them into electrical energy which can be stored and used by the sensor (Ho and Zhang [2012]). For sensors using this technology, the energy (but not the energy-rate) is typically "unlimited" compared to battery-powered sensor as the harvester can generate power all the time during the whole time-horizon. But unlike the battery-powered sensor which has relatively explicit energy amount for future use, the sensor with energy harvester will be subject to an unpredictable future energy level as they are affected by the external environment. Due to the randomness of the amounts of harvested energy in the following time steps, new challenges arise in the design and analysis of the communication strategy of the sensor.

In Nayyar et al. [2013], the authors investigated a remote estimation problem for an energy harvesting sensor and a remote estimator. The communication strategy for the sensor and the estimation strategy for the remote estimator are jointly optimized in terms of the expected sum of communication and distortion costs, again using a dynamic programming approach. Nourian et al. [2013] studied optimal transmission energy allocation scheme for error covariance minimization in Kalman filtering with random packet losses when the sensors have energy harvesting capabilities, and they provided some structural results on the optimal solution for both finite and infinite time-horizon. Different from these works, we specify the different distributions of different environment conditions for the energy harvesting model. Furthermore, we use a smart sensor to pre-processes the measurement data which can improve the estimation quality Hovareshti et al. [2007]. The main challenges and contributions of this work are summarized as follows:

(1) **Randomness of harvested energy:** In previous works, e.g., Li et al. [2013b], the constraints of the transmission power are deterministic. For energy harvesting sensors, on the other hand, the information of the energy constraints is not exactly available for the

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sensor before the harvesting due to the randomness of the energy resources. To handle this new challenge, we develop a new approach.

(2) **Infinite time-horizon MDP:** We consider an infinite time-horizon problem, which is a better approximation for long-run applications and more difficult. In order to overcome the randomness of the energy resources, we prove that an associated power control design problem can be formulated into a standard MDP framework with infinite time-horizon and give the optimal solution.

(3) **Sub-optimal solution:** As the MDP method cannot in general provide an explicit form of the optimal solution and the computational complexity is formidable for general higher-order systems, we propose a sub-optimal solution which is in threshold form and is easy to implement for different system parameter settings.

The remainder of this manuscript is organized as follows. Section 2 presents the system setup. Section 3 formulates the problem into a standard MDP framework and provides the optimal solution. Section 4 introduces a sub-optimal solution and compares it with the optimal one. Numerical example and simulations are included in Section 5. Section 6 draws conclusions.

Notations: $\mathbb{Z}$ denotes the set of integers and $\mathbb{N}$ the positive integers. $\mathbb{R}$ is the set of real numbers. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{S}_+^n$ (and $\mathbb{S}_{++}^n$) is the set of n by n positive semi-definite matrices (and positive definite matrices). When $X \in \mathbb{S}_+^n$ (and $\mathbb{S}_{++}^n$), we write $X \geqslant 0$ (and $X > 0$). $X \geqslant Y$ if $X - Y \in \mathbb{S}_+^n$. $\mathrm{Tr}(\cdot)$ is the trace of a matrix. The superscript $'$ stands for transposition. For functions f, f_1, f_2 with appropriate domains, $f_1 \circ f_2(x)$ stands for the function composition $f_1\big(f_2(x)\big)$, and $f^n(x) \triangleq f\big(f^{n-1}(x)\big)$, where $n \in \mathbb{N}$ and with $f^0(x) \triangleq x$. δ_{ij} is Dirac delta function, i.e., δ_{ij} equals to 1 when $i = j$ and 0 otherwise. The notation $\mathbb{P}[\cdot]$ refers to probability and $\mathbb{E}[\cdot]$ to expectation.

2. STATE ESTIMATION WITH AN ENERGY HARVESTER

We consider the problem of remote estimating the state of the following linear time-invariant (LTI) system:

$$x_{k+1} = Ax_k + w_k, \tag{1}$$
$$y_k = Cx_k + v_k, \tag{2}$$

where $k \in \mathbb{N}$, $x_k \in \mathbb{R}^{n_x}$ is the system state vector at time k, $y_k \in \mathbb{R}^{n_y}$ is the measurement taken by the sensor, $w_k \in \mathbb{R}^{n_x}$ and $v_k \in \mathbb{R}^{n_y}$ are zero-mean i.i.d. Gaussian noises with $\mathbb{E}[w_k w_j'] = \delta_{kj}Q$ $(Q \geqslant 0)$, $\mathbb{E}[v_k(v_j)'] = \delta_{kj}R$ $(R > 0)$, $\mathbb{E}[w_k(v_j)'] = 0$ $\forall j, k \in \mathbb{N}$. The initial state x_0 is a zero-mean Gaussian random vector with covariance $\Pi_0 \geqslant 0$ and is uncorrelated with w_k and v_k. The pair (A, C) is assumed to be observable and $(A, Q^{1/2})$ is controllable.

2.1 Sensor Local State Estimate

We assume the sensor in this work is embedded with an on-board processor (Hovareshti et al. [2007]), the so called "smart sensor". At each time k, the sensor first locally runs a regular Kalman filter to produce the minimum mean-square error (MMSE) estimate of the state x_k based on all the measurements it collects up to time k. It then transmits the local estimate to a remote estimator.

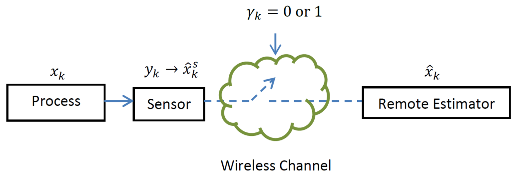

Fig. 1. System Architecture

Denote $\hat{x}_k^s$ and P_k^s as the sensor's local MMSE state estimate and the corresponding estimation error covariance, respectively, i.e.:

$$\hat{x}_k^s = \mathbb{E}[x_k | y_1, y_2, ..., y_k], \tag{3}$$
$$P_k^s = \mathbb{E}[(x_k - \hat{x}_k^s)(x_k - \hat{x}_k^s)' | y_1, y_2, ..., y_k], \tag{4}$$

which can be calculated recursively using standard Kalman filter update equations (Anderson and Moore [1981]), where the recursion starts from $\hat{x}_0^s = 0$ and $P_0^s = \Pi_0 \geqslant 0$.

The following Lyapunov and Riccati operators $h, \tilde{g} : \mathbb{S}_+^n \to \mathbb{S}_+^n$ are introduced to facilitate our subsequent discussion:

$$h(X) \triangleq AXA' + Q, \tag{5}$$
$$\tilde{g}(X) \triangleq X - XC'[CXC' + R]^{-1}CX. \tag{6}$$

Since the estimation error covariance P_k^s converges to a steady-state value exponentially fast (See Anderson and Moore [1981]), without loss of generality, we assume that the Kalman filter at the sensor side has already entered the steady state, i.e., :

$$P_k^s = \overline{P}, \ k \geqslant 1, \tag{7}$$

where $\overline{P}$ is the steady-state error covariance, which is the unique positive semi-definite solution of $\tilde{g} \circ h(X) = X$.

2.2 Wireless Communication Model

Similar to Li et al. [2013b], the local state estimate of the sensor $\hat{x}_k^s$ is transmitted to the remote estimator over an Additive White Gaussian Noise (AWGN) channel using Quadrature Amplitude Modulation (QAM). Denote ω_k as the transmission power for sending the QAM symbol at time k, which will be designed in the following sections. Based on the analysis in Li et al. [2013b], the approximate relationship between the symbol error rate (SER) and ω_k is given by:

$$\mathrm{SER} \approx \exp\Big(-\beta \frac{\omega_k}{N_0 W} \Big). \tag{8}$$

The communication channel is assumed to be time-invariant, i.e., β, N_0, W, are constants during the whole time horizon [1]. In practice, the remote estimator can detect symbol errors via cyclic redundancy check (CRC). Thus taking into account of the SER in the transmission of QAM symbols, a binary random process $\{\gamma_k\}, k \in \mathbb{N}$ can be used to characterize the equivalent communication channel for $\hat{x}_k^s$ between the sensor and the remote estimator, where:

$$\gamma_k = \begin{cases} 1, & \text{if } \hat{x}_k^s \text{ arrives error-free at time } k, \\ 0, & \text{otherwise (regarded as dropout).} \end{cases} \tag{9}$$

[1] For time-variant channels, one can also formulate the problem in a similar way. This is left for future work