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An Observer with Measurement-triggered Jumps for Linear Systems with Known Input ⋆

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Abstract: This paper deals with the estimation of the state of linear time invariant systems for which measurements of the output are available sporadically. An observer with jumps triggered by the arrival of such measurements is proposed and studied in a hybrid systems framework. The resulting system is written in estimation error coordinates and augmented with a timer variable that triggers the event of new measurements arriving. The design of the observer is performed to achieve uniform global asymptotic stability (UGAS) of a closed set including the points for which the state of the plant and its estimate coincide. Furthermore, a computationally tractable design procedure for the proposed observer is presented and illustrated in an example.

Keywords: Observers, hybrid systems, linear systems, linear matrix inequalities, networked control systems

1. INTRODUCTION

State observer design is undoubtedly a difficult problem, with high relevance in applications. Indeed, observers can be employed to obtain an estimation of certain state variables, which are not directly accessible or also to reduce the number of the sensors used in control systems. Many of the most interesting recent applications pertain to controlled systems linked together through data networks. The nature of such networks may often introduce time delays, asynchronism, packages drop-out, and communication channel limitations; see, for example, Lopez Hurtado et al. (2009). Moreover, in modern distributed systems, the communication mechanisms across the network are governed by logic statements, which aim at reducing the required bandwidth over the communication channel; see, for example, Wong and Brockett (1997). Such mechanisms lead to an intermittent availability of the measured variables. In this setting, the classical paradigm of continuously measured variables needs to be reconsidered to face the new challenges induced by data network constraints. Indeed, an observer can employ the measured output only at discrete-time instants, which are *a priori* unknown, that is the estimation algorithm is actually governed by an event-triggered mechanism (see Åström and Bernhardsson (2002) for further details). It is worthwhile to notice that for the periodic sampling case, several solutions are shown in the literature, (see for example Maroni et al. (2000)).

In this paper, we focus on the estimation problem for linear systems where the input injected into the plant is known and the measured output is gathered in an intermittent fashion. Building from the idea in Raff and Allgöwer (2007), we propose an open-loop observer along with a suitable event-triggered updating of the estimated state. Since the evolution of the considered observer exhibits both continuous-time behavior and instantaneous updating, we provide a hybrid model of the observer including the triggering logic. Then, using a Lyapunov function, we propose a condition that guarantees global uniform asymptotic stability of the estimation error as well as robustness with respect to bounded perturbations, in an input-to-state stability sense (see Sontag (1989) and Cai and Teel (2009)). To this end, by relaxing the input-to-state stability Lyapunov condition for hybrid systems proposed by Cai and Teel in Cai and Teel (2009), we exhibit a novel sufficient condition to prove input-to-state stability in presence of persistent jumps. Finally, the obtained condition is turned into a design algorithm for the proposed observer based on the solution of a set of linear matrix inequalities.

The proposed hybrid model allows us to effectively exploit the properties of the time domain of the solutions to the resulting hybrid system, in particular, the persistence of jumps. This feature not only provides a tighter understanding of the system behavior but also enables us to construct a more general Lyapunov function, so as to

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overcome the convexity issues induced by non-uniformity in sampling time, which are also pointed out in Raff and Allgöwer (2007), and, moreover, to characterize the effect of measurement noise via input-to-state stability.

The paper is organized as follows. Section II presents the system under consideration, the problem we intend to solve, and the hybrid modeling of the proposed observer. Section III is dedicated to the main results, which provide a solution to the stated estimation problem. Section IV is devoted to numerical issues and provides a convex design algorithm for the proposed observer. In Section V, the effectiveness of the approach is illustrated through a numerical example. Due to space limitations, proofs of the results will be published elsewhere.

Notation: The set $\mathbb{N}_0$ is the set of the positive integers including zero and $\mathbb{R}_{\geq 0}$ represents the set of the nonnegative real scalars. For every complex number ω, $\mathfrak{Re}(\omega)$ and $\mathfrak{Im}(\omega)$ stand respectively for the real and the imaginary part of ω. $\mathbf{I}$ denotes the identity matrix whereas $\mathbf{0}$ denotes the null matrix (equivalently the null vector) of appropriate dimensions. For a matrix $A \in \mathbb{R}^{n\times m}$, A' denotes the transpose of A and $\|A\|$ denotes the Euclidean induced norm. $\mathrm{He}(A) = A+A'$. For two symmetric matrices, A and B, $A > B$ means that $A-B$ is positive definite. In partitioned symmetric matrices, the symbol $\star$ stands for symmetric blocks. The matrix $\mathrm{diag}\{A_1; \dots; A_n\}$ is the block-diagonal matrix having $A_1, \dots, A_n$ as diagonal blocks. For a vector $x \in \mathbb{R}^n$, x' denotes the transpose of x, whereas $\|x\|$ denotes the Euclidean norm. For a function $s \in [0, +\infty) \to \mathbb{R}^n$, $\|s\|_t = \sup_{\tau\in[0,t]} \|s(\tau)\|$. Let X be a given set, $\mathrm{Co}\{X\}$ represents the convex hull of X. $\delta\mathbb{B}$ is the closed ball with radius δ of appropriate dimension in the Euclidean norm. A function $\alpha\colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to belong to the class $\mathcal{K}$ if it is continuous, zero at zero, and strictly increasing. A function $\alpha\colon \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to belong to class $\mathcal{K}_\infty$ if it belongs to the class $\mathcal{K}$ and is unbounded. A function $\beta\colon \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to belong to class $\mathcal{KL}$ if it is nondecreasing in its first argument, nonincreasing in its second argument, and $\lim_{s\to 0^+} \beta(s,t) = \lim_{t\to+\infty} \beta(s,t) = 0$. A function $\beta\colon \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is said to belong to class $\mathcal{KLL}$ if, for each $r \in \mathbb{R}_{\geq 0}$, the functions $\beta(\cdot,\cdot,r)$ and $\beta(\cdot,r,\cdot)$ belong to class $\mathcal{KL}$.

2. PROBLEM STATEMENT

2.1 System description

Consider the following continuous-time linear system:

$$\begin{aligned} \dot{z} &= Az + Bu \\ y &= Mz \end{aligned} \tag{1}$$

where $z \in \mathbb{R}^n$, $y \in \mathbb{R}^q$ and $u \in \mathbb{R}^p$ are, respectively, the state, the measured output, and the input of the system, while A, B and M are constant matrices of appropriate dimensions. Assume also that the input u belongs to the class of the measurable and locally bounded functions $u\colon [0,\infty) \to \mathbb{R}^p$. We want to design an observer providing an estimate $\hat{z}$ of the state z when the output y is available only at some times t_k, for $k \in \mathbb{N}_0$, not known *a priori*. Figure 1 illustrates such a setting in the context of network control. Suppose that $\{t_k\}_0^{+\infty}$ is a strictly increasing unbounded real sequence of times. Furthermore, assume that there exist two positive real scalars T_1, T_2 with $T_1 < T_2$ such that [1]

[1] Concerning this assumption, see Postoyan et al. (2011); Briat and Seuret (2012) and the references therein. Notice that, as pointed also in Hetel et al. (2012), condition (2) prevents the existence of accumulation points in the sequence $\{t_k\}_0^{+\infty}$, and, hence, it avoids the existence of Zeno behaviors, which is typically undesired in practice.

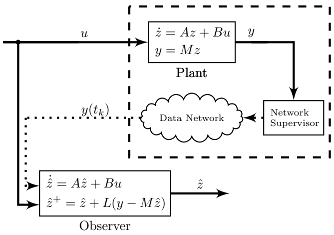

Fig. 1. State estimation for a linear system with output gathered through a data network.

$$T_1 \leq t_{k+1} - t_k \leq T_2. \tag{2}$$

Since the information on the output y is available in an impulsive fashion, motivated by the work of Raff and Allgöwer (2007), to solve the considered estimation problem, we design an observer with jumps in its state following the law:

$$\dot{\hat{z}} = A\hat{z} + Bu \quad \text{when}\, t \notin \{t_k\}_0^{+\infty} \tag{3a}$$

$$\hat{z}(t_k^+) = \hat{z}(t_k) + L(y(t_k) - M\hat{z}(t_k)) \quad \text{when}\, t \in \{t_k\}_0^{+\infty} \tag{3b}$$

where L is a real matrix of appropriate dimensions to be designed. It is worthwhile to point out that in Sur and Paden (1997) the same observer is adopted to state estimation in presence of quantized measurement.

Following the lines of Sanfelice and Praly (2012), the state estimation problem can be formulated as a set stabilization problem. Namely, define

$$\mathcal{A}_s = \left\{(z,\hat{z}) \in \mathbb{R}^{2n} \colon z = \hat{z}\right\} \tag{4}$$

our goal is to design the matrix L such that $\mathcal{A}_s$ is globally asymptotically stable for the plant (1) interconnected with the observer in (3a). At this stage, as usual in estimation problems, one considers the estimation error defined as $\varepsilon := z - \hat{z}$, so the error dynamics are given by the following dynamical system with jumps:

$$\dot{\varepsilon} = A\varepsilon \qquad \text{when}\, t \notin \{t_k\}_0^{+\infty} \tag{5a}$$

$$\varepsilon(t_k^+) = (\mathbf{I} - LM)\varepsilon(t_k) \qquad \text{when}\, t \in \{t_k\}_0^{+\infty}. \tag{5b}$$

Due to the linearity of the system (1), the estimation error dynamics and the dynamics of z are decoupled. Then, for the purpose of stabilizing the set $\mathcal{A}_s$, one can effectively just consider system (5).

Remark 1. Notice that assuming the knowledge of the input is not overly restrictive. Indeed, in many practical settings, all of the devices employed to control and supervise the plant may be embedded into the same system. This situation is depicted in Figure 1, where the dotted arrows denote impulsive data streams, while the solid arrows denote continuous data streams. Notice also that, often, the estimated state is part of a feedback controller (*e.g.* in linear observer-based controller architectures), in which case the input u is a static function of the estimated state that is perfectly known.

2.2 Hybrid modeling

The fact that the observer experiences jumps when a new measurement is available suggests that the updating