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Iterative Learning Control for Periodic Systems using Model Predictive Methods with adaptive sampling rates

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Abstract: This paper addresses iterative learning control (ILC) for periodic systems using model predictive and optimization methods to redesign trajectories and reject periodic disturbances. Stability and optimality of these optimization methods is analysed and illustrated on simulations. The additional prospects of the optimization formulation (e.g. including energy costs, system identification) referred to the trajectory planning are accentuated. To reduce the calculation effort of the optimization algorithm a variable and adaptive sampling period is introduced. The advantages compared to classical ILC methods especially in consideration of constraints are presented.

Keywords: Iterative learning control, Model predictive control, Optimization, Periodic control, Trajectory planning, Target tracking, Stability, Disturbance rejection

1. INTRODUCTION

Iterative Learning Control (ILC) (Arimoto et al. [1984] and Moore [1993]) is widely used in industrial repetitive/periodic and iterative processes (*iterative*: robotic automation systems, machine press; *periodic*: motors with eccentric, oscillating steam engines). In general the main idea of this control concept is the disturbance rejection by adapting the reference trajectories. Classical learning approaches transform the trajectories using e.g. P-controllers (Moore [2001], Ratcliffe et al. [2005])

$$u_{j+1} = u_j + Ke_j \tag{1}$$

where u_j is the trajectory of the current iteration, u_{j+1} is the trajectory of the next iteration, e is the tracking error and K is the gain of the ILC. By saving data from the last cycles, calculating new trajectories and applying them to the system, the control error can be reduced iteratively. More information can be found in the survey papers of iterative learning control Wang et al. [2009] and Bristow et al. [2006]. One of the biggest disadvantages of the classical approaches is the absence of a system/disturbance model (model/predictive information) which could significantly improve the control performance.
Many approaches can be found in the literature to solve these problems. ILC strategies using PD/PID controllers (Chen and Moore [2002]/Park et al. [1999], Madady [2008]) include predictive information for a small horizon (one step).
Anticausal filtering algorithms (Verwoerd [2005], van de Wijdeven and Bosgra [2007]) solve the ILC problem using information from the last iterations. The stored data can be referred to the future system behavior and used for the anticausal filter functions. Due to the filter characteristics these methods are unsuitable for changing initial conditions.
Further approaches discuss optimization methods (Pandit and Buchheit [1999], Lee et al. [2000]) which improve the controlled process using system model information. These methods can be divided into two groups: static optimization and dynamic optimization. In general, both approaches are only applicable to cyclic non-periodic systems (non-changing initial condition). The proposed approach in this paper shows how system model data and in addition system limitations (state/input constraints) can be included very efficiently into an ILC design for periodic processes (changing initial conditions) using the beneficial structure of periodic systems (Section 3).
Model predictive control (MPC) ILC approaches which combine the inner control design (process) with the outer ILC strategy can be found in Lee and Lee [2000], Cueli and Bordons [2008] and Wang and Doyle [2009], Chen et al. [2013]. Therefore, for the inner process a MPC has to be designed such that the ILC concept is included. Due to the system dynamic, the prediction horizon of the MPC is limited (calculation effort). A separation of ILC and control design is not given by these concepts which is contradictory to the general idea of ILC: to formulate a general separable approach for controlled cyclic/periodic processes.
For a separable ILC design for periodic processes under constraints new methods have to be developed. The approach presented in this paper concerns the specified issue outlined above using model predictive methods. For this purpose, a minimization problem is introduced and solved such that the stability of the ILC is guaranteed and the learning rate can be adapted continuously without loss of stability. Including MPC methods into the ILC approach leads to planned trajectories considering system constraints. These trajectories are calculated at the beginning of each period. Adapting the cost function of

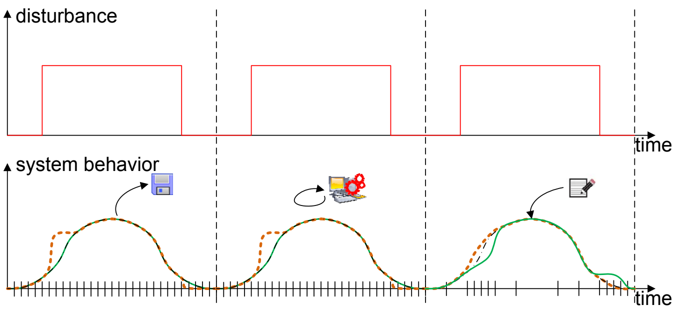

Fig. 1. ILC process

the resulting minimization problem can meet additional optimization objectives (minimization of energy, identification of the system dynamic, adapting inner control parameters). The considered system classes of the approach can be extended to linear time invariant systems with underlying nonlinear systems. To handle large prediction horizons and to reduce the calculation effort, the model predictive strategy uses variable sampling periods. This allows an adaption of the learning process related to the disturbance. In this paper this is called flexible focused learning (FFL).

The paper is organized as follows: Concept and idea of the ILC approach are introduced in Section 2. In Section 3, modelling, optimization, stability and calculation effort of the control concept are presented. In Section 4, an illustrative example is given to demonstrate the prospects of the approach. Finally, Section 5 concludes the paper and accentuates further prospects.

Throughout the paper scalars are indicated by nonbold letters and vectors and matrices by bold letters.

2. CONCEPT

To realize ILC for periodic systems using model predictive methods it is essential to save all state information for one period time. This state information can be used to calculate the optimization step sizes using a step size calculator (reduction of calculation effort). In parallel, the required matrices for the optimization algorithm must be constructed (Section 3). Finally, the optimization problem is solved for the next period and applied to the system. The structure of the ILC process is illustrated in Figure 1 where the system behavior plot shows the reference signal (dash-dotted), the ILC output (solid) and the system response (dashed). For small tracking errors or rather small disturbance variations a large step size is reasonable. If large tracking errors occur, a small step size has to be used around the local tracking error (FFL). The width of the region around the local error is set according to the largest system time constant and the weighting matrices of the optimization function. To keep the ILC concept general, the approach generates trajectories for all state variables (as needed for state control, flatness based control or nonlinear control techniques).

In this paper the method will be illustrated on a state space controlled third order LTI system described by

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}u + \boldsymbol{B}_{\mathrm{d}}d \quad (2)$$

where $\boldsymbol{x} \in \mathbb{R}^3$ is the state vector, $u \in \mathbb{R}$ is the input and $d \in \mathbb{R}$ is the disturbance. The system is controllable and observable. For simplicity, the approach is described

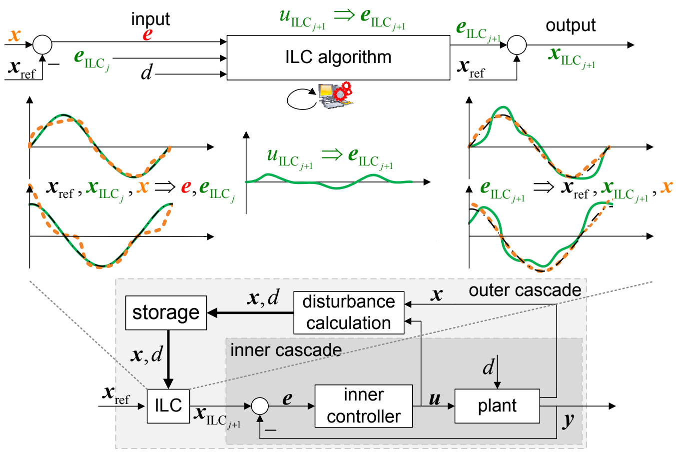

Fig. 2. ILC structure and control structure

on a single input system. Nevertheless, the theory is also applicable to n-order MIMO systems. The state space controlled process of the example (with ILC) referred to the reference trajectory $\boldsymbol{z}_{\mathrm{nref}}$ can be reformulated to

$$\dot{\boldsymbol{z}}_{\mathrm{n}} = \boldsymbol{A}_{\mathrm{zn}}\boldsymbol{z}_{\mathrm{n}} + \boldsymbol{B}_{\mathrm{zn}}\underbrace{\left[\boldsymbol{a}_{\mathrm{n}}^T\ 1\right]\begin{bmatrix}\boldsymbol{z}_{\mathrm{nref}}+\boldsymbol{e}_{\mathrm{znILC}}\\ \dot{z}_{\mathrm{n}_n\mathrm{ref}}+\dot{e}_{\mathrm{zn}_n\mathrm{ILC}}\end{bmatrix}}_{u_{\mathrm{nref}}+u_{\mathrm{nILC}}=\left[\boldsymbol{a}_{\mathrm{n}}^T\ 1\right]\boldsymbol{z}_{\mathrm{nILC}}} + \boldsymbol{B}_{\mathrm{zdn}}d_{\mathrm{n}}. \quad (3)$$

Here, the system is transformed to the controllable canonical form (CCF) with the CCF-coefficients $-\boldsymbol{a}_{\mathrm{n}}^T$ and normalized (with $\boldsymbol{S} = \mathrm{diag}(\boldsymbol{s})$ and the normalization coefficients $\boldsymbol{s}$) to $\boldsymbol{z}_{\mathrm{n}} = \boldsymbol{S}_{\mathrm{z}}\boldsymbol{z}$, $\boldsymbol{A}_{\mathrm{zn}} = \boldsymbol{S}_{\mathrm{z}}\boldsymbol{A}_{\mathrm{z}}\boldsymbol{S}_{\mathrm{z}}^{-1}$, $\boldsymbol{B}_{\mathrm{zn}} = \boldsymbol{S}_{\mathrm{z}}\boldsymbol{B}_{\mathrm{z}}\boldsymbol{S}_{\mathrm{u}}^{-1}$ and $\boldsymbol{B}_{\mathrm{zdn}} = \boldsymbol{S}_{\mathrm{z}}\boldsymbol{B}_{\mathrm{zd}}\ \boldsymbol{S}_{\mathrm{d}}^{-1}$ which is required due to the ILC approach and due to the numerical precision of the algorithm. In addition, a normalization leads to comparable weighting matrices of the optimization problem (Section 4). $u_{\mathrm{nILC}}|\boldsymbol{e}_{\mathrm{znILC}}$ is the additional ILC reference trajectory. $\boldsymbol{z}_{\mathrm{nILC}}$ is the resultant reference trajectory.

In general, the system tracking error dynamic of the periodic process is crucial for the ILC. Hence, the state space dynamic must be referred to. For the presented example, the error dynamic is given by (using Eq. (3))

$$\dot{\boldsymbol{e}}_{\mathrm{n}} = \boldsymbol{A}_{\mathrm{zn}}\boldsymbol{e}_{\mathrm{n}} + \boldsymbol{B}_{\mathrm{zn}}\underbrace{\left[\boldsymbol{a}_{\mathrm{n}}^T\ 1\right]\begin{bmatrix}\boldsymbol{e}_{\mathrm{znILC}}\\ \dot{e}_{\mathrm{zn}_n\mathrm{ILC}}\end{bmatrix}}_{u_{\mathrm{nILC}}} + \boldsymbol{B}_{\mathrm{zdn}}d_{\mathrm{n}}. \quad (4)$$

The calculated additional reference trajectory of the ILC optimization is given by u_{nILC}. To calculate the corresponding values $\boldsymbol{e}_{\mathrm{znILC}}$, the system dynamic equations

$$\begin{aligned}\dot{\boldsymbol{e}}_{\mathrm{zn}} &= \boldsymbol{A}_{\mathrm{zn}}\boldsymbol{e}_{\mathrm{zn}} + \boldsymbol{B}_{\mathrm{zn}}u_{\mathrm{nILC}} + \boldsymbol{B}_{\mathrm{zdn}}d_{\mathrm{n}}\\ \dot{\boldsymbol{e}}_{\mathrm{znwo}} &= \boldsymbol{A}_{\mathrm{zn}}\boldsymbol{e}_{\mathrm{znwo}} + \boldsymbol{B}_{\mathrm{zdn}}d_{\mathrm{n}}.\end{aligned} \quad (5)$$

are needed. To get $\boldsymbol{e}_{\mathrm{nILC}}$, the results have to be subtracted.

$$\boldsymbol{e}_{\mathrm{znILC}} = \boldsymbol{e}_{\mathrm{zn}} - \boldsymbol{e}_{\mathrm{znwo}} \quad (6)$$

For brevity, the proof is omitted. The calculation of $\dot{e}_{\mathrm{zn}_n\mathrm{ILC}}$ results from the derivative of $e_{\mathrm{n}_n\mathrm{ILC}}$. Figure 2 illustrates the ILC structure and the control structure. For the ILC structure (upper figure) the transformation, normalization and state vector computation is done inside the ILC algorithm. j indicates the current period cycle, $\boldsymbol{x}_{\mathrm{ref}}$ is the reference state vector, $\boldsymbol{e}$ is the tracking error, u_{ILC} or rather $\boldsymbol{e}_{\mathrm{ILC}}$ is the additional ILC reference trajectory and $\boldsymbol{x}_{\mathrm{ILC}}$ is the resultant reference trajectory of the system. The control structure (lower figure) shows the relations