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Disturbance Decoupling with Stability in Continuous-Time Switched Linear Systems Under Dwell-Time Switching

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Abstract: This work deals with state feedback compensation of disturbance inputs in continuous-time switched linear systems, with the requirement that the closed-loop systems be exponentially stable under switching signals with a sufficiently large dwell-time. Constructive conditions for the problem to be solvable are shown, on the assumption that the given switched linear system has zero initial state. The effects of nonzero initial states are inspected. The theoretical background consists of both classic and novel ideas of the geometric approach, enhanced with notions specifically oriented to switched linear systems.

Keywords: switched linear systems; disturbance decoupling; exponential stability; dwell-time.

1. INTRODUCTION

In the last few decades, switched systems have effectively been employed in solving control problems that involve systems with different modes of operation: e.g., LQR optimal control (Balandat et al., 2012), $\mathcal{H}_2$ control (Mahmoud, 2009), $\mathcal{H}_\infty$ control (Deaecto et al., 2011), output regulation (Zattoni et al., 2013), model matching (Conte et al., 2014), and disturbance decoupling (Otsuka, 2010; Conte and Perdon, 2011; Zattoni and Marro, 2013) are typical synthesis problems recently formulated for switched systems. As to disturbance decoupling, the abovementioned papers are focused on the requirement that the closed-loop system be quadratically stable. In (Otsuka, 2010; Conte and Perdon, 2011), quadratic stability of the closed-loop system is sought for a suitable switching law. In (Zattoni and Marro, 2013), quadratic stability is requested for arbitrary switching signals. However, quadratic stability is quite a demanding specification. As is well-known (e.g., Lin and Antsaklis, 2009), quadratic stability under arbitrary switching is only a sufficient condition for asymptotic stability and could be rather restrictive. Moreover, it has also been shown that switched systems may not be asymptotically stable under arbitrary switching, but may enjoy this property for some classes of switching signals, satisfying specific constraints. In addition, restrictions on the switching signals may arise from physical constraints on the systems or may be inferred from some knowledge of the switching rules. For these reasons, in this work, we will investigate the problem of disturbance decoupling with exponential stability under restricted switching.

Notation: $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{Z}^+$, and $\mathbb{C}^-$ stand for the sets of real numbers, nonnegative real numbers, nonnegative integer numbers, and complex numbers with negative real part, respectively. Matrices and linear maps are denoted by upper-case letters, like A. The image, the kernel, and the spectrum of A are denoted by $\operatorname{im} A$, $\ker A$, and $\lambda(A)$, respectively. The transpose of A is denoted by $A^\top$. Vector spaces and subspaces are denoted by calligraphic letters, like $\mathcal{V}$. The quotient space of a subspace $\mathcal{V}$ over a subspace $\mathcal{W} \subseteq \mathcal{V}$ is denoted by $\mathcal{V}/\mathcal{W}$. The restriction of a linear map A to an A-invariant subspace $\mathcal{J}$ is denoted by $A|_\mathcal{J}$. The inverse image of a subspace $\mathcal{V}$ through a linear map B is denoted by $B^{-1}\mathcal{V}$. The symbol $\uplus$ denotes union with repetition count. The symbols I and O respectively stand for an identity matrix and a zero matrix with appropriate dimensions.

2. PROBLEM STATEMENT

Let $\Sigma_{\sigma(t)}$ be a continuous-time switched linear system defined by

$$\Sigma_{\sigma(t)} \equiv \begin{cases} \dot{x}(t) = A_{\sigma(t)}\, x(t) + B_{\sigma(t)}\, u(t) + H_{\sigma(t)}\, h(t), \\ e(t) = E_{\sigma(t)}\, x(t), \end{cases} \tag{1}$$

where $t \in \mathbb{R}^+$ is the time variable, $x \in \mathcal{X} = \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ is the control input, $h \in \mathbb{R}^m$ is the disturbance input, and $e \in \mathbb{R}^q$ is the output, with $p, m, q \leq n$. Let the modes of $\Sigma_{\sigma(t)}$ be the linear time-invariant systems of the set $\{\Sigma_i,\, i \in \mathcal{I}\}$, where $\mathcal{I} = \{1, 2, \ldots, N\}$ and

$$\Sigma_i \equiv \begin{cases} \dot{x}(t) = A_i\, x(t) + B_i\, u(t) + H_i\, h(t), \\ e(t) = E_i\, x(t), \end{cases} \quad i \in \mathcal{I}, \tag{2}$$

with A_i, B_i, H_i, E_i constant real matrices of suitable dimensions. Let B_i, H_i, E_i be full-rank matrices. Let the sets of the admissible control input signals and of the admissible disturbance input signals be respectively defined as the sets of piecewise-continuous functions $u(t)$ and $h(t)$, with $t \in \mathbb{R}^+$, taking finite values in $\mathbb{R}^p$ and $\mathbb{R}^m$.

Let the switching signal $\sigma(t)$ be defined as a measurable and not a-priori known map $\sigma:\mathbb{R}^+\to\mathcal{I}$, $t\to i$, so that the active mode at the time $t\in\mathbb{R}^+$ is Σ_i, with $i=\sigma(t)$. The switching signal $\sigma(t)$ is assumed to be subject to time-domain restrictions as specified below. Let t_ℓ, with $\ell\in\mathbb{Z}^+$, be the sequence of the switching times. The positive real constant τ, defined as $\tau=\inf_{\ell\in\mathbb{Z}^+}\{t_{\ell+1}-t_\ell\}$, is assumed to be greater than or equal to a finite positive real constant τ_d. The set of all switching signals $\sigma(t)$ with τ no smaller than τ_d is denoted by $\mathscr{S}_{\tau_d}$ and the finite positive real constant τ_d is called dwell-time. Hence, the time-domain restriction on $\sigma(t)$ can be concisely expressed as $\sigma(t)\in\mathscr{S}_{\tau_d}$.

Let $F_{\sigma(t)}$ denote a switched state feedback, associated with the set $\{F_i\in\mathbb{R}^{p\times n},\ i\in\mathcal{I}\}$. Hence, the closed-loop system is described by the continuous-time switched linear system

$$\hat{\Sigma}_{\sigma(t)}\equiv\begin{cases}\dot{x}(t)\,=\,(A_{\sigma(t)}+B_{\sigma(t)}\,F_{\sigma(t)})\,x(t)+H_{\sigma(t)}\,h(t),\\ e(t)\,=\,E_{\sigma(t)}\,x(t),\end{cases}\tag{3}$$

with the modes

$$\hat{\Sigma}_i\equiv\begin{cases}\dot{x}(t)\,=\,(A_i+B_i\,F_i)\,x(t)+H_i\,h(t),\\ e(t)\,=\,E_i\,x(t),\end{cases}\quad i\in\mathcal{I}.\tag{4}$$

Let the following assumption hold:

$\mathcal{A}\,1$. $x(0)=0$.

Assumption $\mathcal{A}\,1$ is a standing assumption in perfect decoupling problems. However, as will be observed in Remark 37, if the initial state is different from zero, zero output can still be guaranteed, provided that the initial state belongs to a certain subspace, which will be determined precisely. Moreover, as will be pointed out in Remark 38, if the initial state is different from zero and does not belong to the abovementioned subspace, asymptotic decoupling can be achieved in place of perfect decoupling, provided that suitable stability conditions are satisfied.

The problem of disturbance decoupling, with the requirement that the closed-loop system be exponentially stable under dwell-time switching, is stated as follows.

Problem 1. Given the continuous-time switched linear system $\Sigma_{\sigma(t)}$, defined by (1), with the modes $\{\Sigma_i,\ i\in\mathcal{I}\}$, defined by (2), find a switched state feedback $F_{\sigma(t)}$, associated with the set $\{F_i,\ i\in\mathcal{I}\}$, such that, on Assumption $\mathcal{A}\,1$, the following requirements are satisfied:

- $\mathcal{R}\,1$. the output $e(t)$ be equal to zero for all $t\in\mathbb{R}^+$, for any admissible disturbance $h(t)$, with $t\in\mathbb{R}^+$;
- $\mathcal{R}\,2$. the system $\hat{\Sigma}_{\sigma(t)}$, defined by (3), with the modes $\{\hat{\Sigma}_i,\ i\in\mathcal{I}\}$, defined by (4), be exponentially stable over $\mathscr{S}_{\tau_d}$, for some finite positive real constant τ_d.

3. GEOMETRIC APPROACH FOR SWITCHED LINEAR SYSTEMS

The purpose of this section is to gather the notions of the geometric approach that will be used to solve Problem 1. For the reader's convenience, some basic concepts are reviewed (Basile and Marro, 1992; Wonham, 1985). Novel geometric objects, like the reachability subspaces constrained to the maximal robust controlled invariant subspace, and new geometric ideas, like those of internal and external exponential stabilizability of the maximal robust controlled invariant subspace under dwell-time switching, are also introduced.

The definitions and properties surveyed below refer to the continuous-time switched linear system $\Sigma_{\sigma(t)}$, defined by (1), with the modes $\{\Sigma_i,\ i\in\mathcal{I}\}$, defined by (2). Short notations for images and null spaces of input and output matrices, respectively, are used: $\mathcal{B}_i=\operatorname{im}B_i$, $\mathcal{H}_i=\operatorname{im}H_i$, and $\mathcal{E}_i=\ker E_i$, with $i\in\mathcal{I}$. The subspace $\mathcal{E}\subseteq\mathcal{X}$ is defined by $\mathcal{E}=\bigcap_{i\in\mathcal{I}}\mathcal{E}_i$. A subspace $\mathcal{J}\subseteq\mathcal{X}$ is said to be a robust A_i-invariant subspace if $A_i\,\mathcal{J}\subseteq\mathcal{J}$, for all $i\in\mathcal{I}$. A subspace $\mathcal{V}\subseteq\mathcal{X}$ is said to be a robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace if $A_i\,\mathcal{V}\subseteq\mathcal{V}+\mathcal{B}_i$, for all $i\in\mathcal{I}$. A subspace $\mathcal{V}\subseteq\mathcal{X}$ is a robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace if and only if there exists a set of linear maps $\{F_i,\ i\in\mathcal{I}\}$, such that $(A_i+B_i\,F_i)\,\mathcal{V}\subseteq\mathcal{V}$, for all $i\in\mathcal{I}$.

As was first shown in (Basile and Marro, 1987), the set of all robust $(A_i,\mathcal{B}_i)$-controlled invariant subspaces contained in a given subspace $\mathcal{E}$ is an upper semilattice, with the sum as binary operation and the inclusion as partial ordering relation. The maximum of the set of all robust $(A_i,\mathcal{B}_i)$-controlled invariant subspaces contained in the subspace $\mathcal{E}$ is called the maximal robust $(A_i,\mathcal{B}_i)$-controlled invariant subspace contained in $\mathcal{E}$ and is denoted by $\mathcal{V}_R^*$. A double-recursion algorithm for computing $\mathcal{V}_R^*$ was also given in (Basile and Marro, 1987, Algorithm 1).

The remainder of this section is split into two parts. Section 3.1 is aimed at introducing the notions of internal switched dynamics and internal exponential stabilizability under dwell-time switching of the maximal robust controlled invariant subspace. The purpose of Section 3.2 is introducing the notions of external switched dynamics and external exponential stabilizability under dwell-time switching of the same subspace.

3.1 Internal Switched Dynamics and Internal Exponential Stabilizability Under Dwell-Time Switching of the Maximal Robust Controlled Invariant Subspace

In this work, the notion of maximal robust controlled invariant subspace contained in a given subspace is referred to the modes of a switched linear systems. Hence, switched dynamics can be induced on that subspace and stabilizability issues can be raised for those dynamics. This section is centred on the definition of internal switched dynamics and the property of exponential stabilizability under dwell-time switching of such dynamics. The exponential stabilizability under dwell-time switching of the internal dynamics of $\mathcal{V}_R^*$ depends on the properties of the fixed internal dynamics of $\mathcal{V}_R^*$ with respect to each system of the set $\{\Sigma_i,\ i\in\mathcal{I}\}$. In order to analyze this aspect in detail, the reachability subspace constrained to $\mathcal{V}_R^*$ — henceforth denoted by $\mathcal{R}_{\mathcal{V}_R^*,i}$ — is introduced for each system Σ_i. Hence, the assignable and fixed internal dynamics of $\mathcal{V}_R^*$ with respect to Σ_i can easily be singled out, since the assignable internal dynamics of $\mathcal{V}_R^*$ with respect to Σ_i coincides with the internal dynamics of $\mathcal{R}_{\mathcal{V}_R^*,i}$. Based on this fact and on a sufficient condition for a switched linear dynamics to be exponentially stable under dwell-time switching (Morse, 1996), a sufficient condition for the internal switched dynamics of $\mathcal{V}_R^*$ to be exponentially stabilizable under dwell-time switching is given. It is worth mentioning that the