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The Information Structure of Feedforward/Preview Control Using Forecast Data

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Abstract: Preview control using a fedforward imperfect forecast measurement of a disturbance signal is investigated in the context of discrete-time linear quadratic Gaussian (LQG) control. A new approach for incorporating such forecast measurements is built directly on established preview control models and results. The calculation of the optimal control gain, for which an efficient computation has already been derived, is found to be independent of the stochastic forecast measurements, implying that the optimal state estimator is where performance improvements in this problem set-up occur. Most significantly, the forecast data model is shown to equip the problem with a nested information structure whereby any forecast feedforward control problem of a fixed horizon length is always equivalent to a problem with a longer horizon and infinitely unreliable forecast measurements beyond the smaller horizon length. A numerical example illustrates the effect of forecast horizon length and data quality on the closed-loop system performance.

Keywords: Feedforward control; preview control; LQG; state estimation; forecasting

1. INTRODUCTION

We formulate the feedforward problem using the (by now) traditional technique of including a delay line, fed by future disturbance values, into an augmented plant structure. This approach dates back, at least, to Tomizuka and Whitney [1975].

We then pose a standard linear quadratic Gaussian (LQG) control problem using this structure, which incorporates the feedforward into the state feedback. This follows closely the $\mathscr{H}_2$-optimal method of Hazell and Limebeer [2010] and yields an LQ feedback gain and a state estimator, along with a closed-loop performance calculation. The separation theorem shows that the LQ gain and the state estimator are designed separately.

The central novelty of the paper rests in the incorporation of separate measurement noises into the feedforward signal, which consists of the entire state of the delay block and not just of its input, as in earlier treatments. This is able to capture, via the associated measurement noise covariances, the forecast phenomenon of diminishing reliability with preview horizon. This is our method for addressing the information structure.

From the linear Gaussian formulation, we prove a well-known (but perhaps unproven) feature that the absence of data can be accommodated through taking infinite variance of that data's measurement noise. This is used to prove, by construction, that the N-step-ahead feedforward controller with forecast information can be precisely embedded within the $(N + k)$-step-ahead feedforward (for $k \geq 0$) with the final k steps having infinite variance measurement noise. This approach also simply addresses the presence of both previewed and unpreviewed disturbances acting on a controlled system.

1.1 Literature

Feedforward or preview control deals with the application of measurements in advance of a disturbance process impinging on a regulated system. These advance measurements are incorporated into the feedback control signal to aid in the rejection of the effects of the disturbance. At its core, feedforward deals with information in control. In this paper we explore this information structure in detail for the case of discrete-time Linear Quadratic Gaussian (LQG) or $\mathscr{H}_2$ control. The study is motivated by control issues in the so-called *Smart Grid*, such as demand response and consumption forecasting, where data from the grid and/or from the external environment (such as weather and irradiance) provide information regarding the demand. A feature of this data is that its quality often varies with horizon of availability. Thus, one-hour-in-advance weather predictions are inherently less reliable than five-minute-in-advance values. Our analysis seeks to explore how such data quality issues can be incorporated into the calculation of feedforward control and, more importantly, how their quality (or lack thereof) affects eventual regulation performance. In this fashion, the results should prove useful for examining the possible impact of capital expenditure on improving the quality of measurements in advance.

Technically, the paper demonstrates that, for LQG control, the information aspects are captured by the state estimator and hence both the feedforward control horizon and the data quality can be divorced from the state feedback gain calculation entirely. This separates the consideration of the informational data properties into just the development of the appropriate Kalman filter. Our approach is to demonstrate that the LQG feedforward control signal with horizon N can be constructed as that of horizon $M \geq N$ and with a related but distinct information structure; the state estimator changes, but all the feedback gains except the M^{th} value remain fixed and this terminal value (as noted by Hazell and Limebeer [2010]) tends to zero exponentially with M. Once this is established, the analysis of the effect of data quality on the performance of LQG feedforward control can take place through the analysis of the state estimator alone.

We rely on four recent lucid papers dealing with the formulation of feedforward and preview control for: LQG systems Hazell and Limebeer [2010] and Roh and Park [1999], Model Predictive Control for linear systems with constraints Carrasco and Goodwin [2011], and $\mathscr{H}_\infty$ control Hazell and Limebeer [2008]. Each of these papers provides a survey of the literature in the field and we shall draw from particularly Hazell and Limebeer [2010] for the underlying problem formulation. Since we are dealing with linear systems, we omit the consideration of reference tracking aspects. For simplicity, the term *feedforward control* will refer to a control law which relies on both a feedforward measurement of the disturbance signal and a feedback measurement from the output of the plant.

The paper unfurls as follows. The underlying problem statement is developed in Section 2, where we adopt the construction from Roh and Park [1999] and Hazell and Limebeer [2010] where the plant model is augmented by a set of delay elements acting on the disturbance before it reaches the plant output. In section 3, feedforward data is incorporated into a standard LQG design and analysis through the provision of measurements from some of the upstream delays. In this section we also describe the informational properties of the feedforward data, notably that of multiple-horizon forecast data, through the incorporation of measurement noise processes into the LQG design. Unlike Hazell and Limebeer [2010], this covers both fedforward and unfedforward disturbance channels in the same breath. The formulation of LQ feedback gain, Kalman state estimator, and LQG performance follow directly. Section 4 constructs the explicit solution of the state feedback gain matrix of feedforward control for horizon N and specializes and improves the solution properties from Hazell and Limebeer [2010]. This is followed in Section 5 by the explicit solution of the associated state estimator with a given information structure and culminates in the demonstration that the N-step-ahead feedforward control signal can be generated by the M-step-ahead solution with any $M \geq N$ and the appropriate information structure. This is the core theoretical analysis of the paper and permits the restriction of the consideration of information structure for feedforward to the design of the state estimator alone. Section 6 provides a brief numerical example.

The contribution of this paper is to provide the analysis of informational aspects of feedforward control, exploiting forecast data, which were not evident in the solution derived in Hazell and Limebeer [2010], where the dependence of disturbance rejection control on the feedforward horizon of exact disturbance data is the prime focus and were not explored in Roh and Park [1999], where only a single noise-corrupted disturbance value is fedforward, as opposed to an entire forecast affected by additive noises of possibly non-uniform covariances. For this current paper, the effect of the data quality of a forecast measurement is of paramount interest. This has not been studied earlier and provides insight into the performance effects of improvements in feedforward data. This information-centric view of feedforward control admits new insights into the solution structure and into the role of the horizon, notably with the application of imperfect forecast data.

2. PROBLEM DESCRIPTION

The system depiction in Figure 1 below contains three subsystems: the plant G, the disturbance model G_d, and the disturbance delay line block G_Δ with discrete-time state-space realizations

$$G := \left[\begin{array}{c|cc} A & B & I \\ \hline C & 0 & 0 \end{array}\right], \quad G_d := \left[\begin{array}{c|c} A_d & B_d \\ \hline C_d & 0 \end{array}\right], \quad G_\Delta := \left[\begin{array}{c|c} A_\Delta & B_\Delta \\ \hline C_\Delta & 0 \\ I & 0 \end{array}\right],$$

where the disturbance model G_d is assumed to be stable.

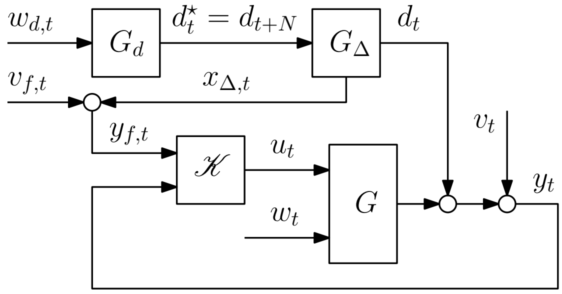

Fig. 1. A feedforward regulator problem with forecast data.

All noises are assumed stationary. The plant output is corrupted by an additive disturbance $d_t \in \mathbb{R}^p$ and an unmeasured, additive measurement noise $v_t \in \mathbb{R}^p$. This new output, $y_t \in \mathbb{R}^p$, is then fedback to controller block $\mathscr{K}$, which also contains an estimator. Future disturbance $d_t^\star = d_{t+N}$ is the result of Gaussian white noise $w_{d,t} \in \mathbb{R}^{m_d}$ feeding into the known system G_d. The current disturbance d_t is generated when $d_t^\star$ is fed into the N-step delay block G_Δ, the state of which $x_{\Delta,t} \in \mathbb{R}^{Np}$ is the sequence of current and future disturbances up to horizon length N.

$$x_{\Delta,t} = \left[d_t^T \ d_{t+1}^T \ \dots \ d_{t+N-1}^T\right]^T.$$

At time t, a preview or forecast of the disturbance, d_{t+n}, is available for $n = 0, 1, \dots, N-1$ in the form of $y_{f,t} \in \mathbb{R}^{Np}$. The reliability of this forecast diminishes with the advancing horizon of the data, i.e. with increasing n. This is incorporated into our model through the inclusion of additive measurement noise $v_{f,t} \in \mathbb{R}^{Np}$ onto the forecast signal , which includes the whole state of the delay line instead of its input signal d_{t+N} as is typically done in preview control.

Hence, the preview signal available to the controller, $\mathscr{K}$, is

$$y_{f,t} = x_{\Delta,t} + v_{f,t},$$

where $v_{f,t}$ is assumed zero mean, white, and Gaussian with

$$\text{cov}(v_{f,t}) = \text{blockdiag}\left[V_{f,0}, V_{f,1}, \dots, V_{f,N-1}\right], \qquad (1)$$

and $V_{f,j} \in \mathbb{R}^{p\times p}$.

$$V_{f,0} \leq V_{f,1} \leq \dots \leq V_{f,N-1}. \qquad (2)$$

The delay structure is captured by taking

$$A_\Delta = \begin{bmatrix} 0 & I & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & I \\ 0 & 0 & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{Np\times Np}, \quad B_\Delta = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I \end{bmatrix} \in \mathbb{R}^{Np\times p},$$

$$C_\Delta = [I \ 0 \ \cdots \ 0] \in \mathbb{R}^{p\times Np},$$

with $I, 0 \in \mathbb{R}^{p\times p}$. We denote the state of the plant G and disturbance model G_d as $x_t \in \mathbb{R}^n$ and $x_{d,t} \in \mathbb{R}^{n_d}$ respectively. $w_t \in \mathbb{R}^n$ is an unmeasured, additive process noise on the plant.

Our approach, as in Hazell and Limebeer [2010], is to apply Linear Quadratic Gaussian (LQG) control to this problem and to develop the controller information architecture by studying the separation into optimal state-variable feedback for a given performance criterion and the optimal state estimation. Specifically, we demonstrate that the LQG solution for this problem possesses an underlying structure where the entire informational aspects of the control reside completely within the estimator design and the state-variable feedback remains fixed. This fixed decomposition holds even when the horizon of the forecast changes. While separation is a well understood aspect of LQG, the existence of a horizon-independent decomposition