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Continuous observer design for nonlinear systems with sampled and delayed output measurements ⋆

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Abstract: We design observers for nonlinear systems with sampled and delayed output measurements. The observers are of continuous and hybrid in nature. Based on an auxiliary integral technique, the exponential stability of the estimation errors is achieved, and the sampling period and the maximum delay are also given. Finally, numerical examples are provided to illustrate the design methods.

1. INTRODUCTION

In this paper, we consider the following system

$$\begin{cases} \dot{x}_1(t) = x_2(t) + f_1(x_1(t)), \\ \dot{x}_2(t) = x_3(t) + f_2(x_1(t), x_2(t)), \\ \qquad \vdots \\ \dot{x}_{n-1}(t) = x_n(t) + f_{n-1}(x_1(t), x_2(t), \cdots, x_{n-1}(t)), \\ \dot{x}_n(t) = f_n(x_1(t), x_2(t), \cdots, x_n(t)) + u(t), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

where the state $x(t) \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}$. We assume that the output $y(t)$ is sampled at instants t_k and is available for the observer at instants $t_k + \tau_k$, where $\{t_k\}$ denotes a strictly increasing sequence such that $\lim_{t\to\infty} t_k = \infty$, and $\tau_k \geq 0$ represents the transmission delay. The sampling interval $T = t_{k+1} - t_k$ is a positive constant. The transmission delays τ_k are unknown, but have an upper bound $\bar{\tau}$, that is, $\max\{\tau_k\} \leq \bar{\tau}$ for all $k = 0, 1, \cdots, \infty$. We also make the assumption: $\bar{\tau} \leq T$, that is, the measures sampled at instants t_k are available for the observer before the next measures sampled at instants t_{k+1}. In addition, $f_i(\cdot)$ $(i = 1, \cdots, n)$ satisfy the following globally Lipschitz condition

$$\begin{aligned} &| f_i(x_1, x_2, \cdots, x_i) - f_i(\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_i) | \\ \leq\ & l(| x_1 - \hat{x}_1 | + | x_2 - \hat{x}_2 | + \cdots + | x_i - \hat{x}_i |), \end{aligned} \tag{2}$$

where l is a positive constant.

There are some results on observer design for (1) without sampled and time delayed measurements, for example, (Deaza [1991], Gauthier et al [1992], Gauthier and Kupka [1994], Praly [2003], Andrieu et al [2009], Shen and Xia [2008], Shen and Huang [2009], Li et al [2013a,b]). In recent years, the observer design for continuous nonlinear systems with sampled and delayed measurements have attracted more and more attention. For linear systems, observers can be designed based on the discrete time model of the continuous systems. However, the design method cannot be extended to nonlinear systems because it is difficult to obtain the exact discrete time model. In such a case, two main approaches are proposed to deal with this problem. The first one is to design a discrete observer by introducing a consistent approximation of the exact discretized model (Arcak and Nešić [2004]). The second one is based on a mixed continuous and discrete design. For instance, a high gain exponential observer is presented in (Deza and Busvelle [1992]). In (Raff et al [2008]), the observers for Lipschitz nonlinear continuous time systems with nonuniformly sampled measurements are designed based on LMI and sampled data control techniques. In (Ahmed-Ali and Lamnabhi-Lagarrigue [2012]), some conditions on the maximum allowable transmission interval are given to guarantee an exponential convergence of the observation error for networked control systems. The authors proposed a sampled-data nonlinear observer design by using a continuous-time design coupled with an inter-sample output predictor (Karafyllis and Kravaris [2009]). In (Nadri et al [2013]), the problem of observer design was investigated for uniformly observable systems with sampled output measurements. There are some results on observer design for nonlinear uniformly observable systems with time-varying delayed output measurement (Van Assche et al [2011], Ahmed-Ali et al [2013a,b]). For example, in (Ahmed-Ali et al [2013b]), the authors addressed two classes of global exponential observers of continuous systems with sampled and delayed output measurements. There are two parts: a prediction part and a correction part. Sufficient conditions on the maximum allowable delay and the maximum allowable sampling period are also given to ensure exponential stability of the observation error.

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In this paper, we consider high gain observer design for the system (1) with sampled and delayed measurements. The observer is continuous and hybrid. By using an auxiliary integral technique, sufficient conditions are presented to ensure the global exponential stability of the observation error. Compared with the existing results, the systems we considered are more general, and the design method is simpler.

This paper is organized as follows. In Section 2, we present our main results: the continuous observer is designed for nonlinear systems with sampled and delayed output measurements. In Section 3, two examples are used to illustrate the validity of the proposed design method. Finally, the paper is concluded in Section 4.

2. CONTINUOUS OBSERVER DESIGN FOR THE SYSTEM (1)

We will design a continuous observer for the system (1), and present the upper bound of the maximum allowable sample period and the delay, so that the observation errors are globally exponentially convergent. The following lemma is useful for our main results.

Lemma 1. (Liu et al [2006]) For any positive definite matrix $M \in \mathbb{R}^{n\times n}$, scalar $\gamma > 0$, vector function $\omega : [0,\gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:

$$\left[\int_0^\gamma w(s)ds\right]^\top M \left[\int_0^\gamma w(s)ds\right] \le \gamma \left[\int_0^\gamma w(s)^\top M w(s)ds\right].$$

For the system (1), we design the following observer,

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + La_1 e_1(t_k) + f_1(\hat{x}_1(t)), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) + L^2 a_2 e_1(t_k) + f_2(\hat{x}_1(t), \hat{x}_2(t)), \\ \quad \vdots \\ \dot{\hat{x}}_{n-1}(t) = \hat{x}_n(t) + L^{n-1} a_{n-1} e_1(t_k) \\ \qquad + f_{n-1}(\hat{x}_1(t), \hat{x}_2(t), \cdots, \hat{x}_{n-1}(t)), \\ \dot{\hat{x}}_n(t) = L^n a_n e_1(t_k) + f_n(\hat{x}_1(t), \hat{x}_2(t), \cdots, \hat{x}_n(t)) \\ \qquad + u(t), \\ \qquad t \in [t_k + \tau_k,\ t_k + T + \tau_{k+1}), \quad k \ge 0, \end{cases} \tag{3}$$

where $L \ge 1$ is a constant, $e_1(t_k) = x_1(t_k) - \hat{x}_1(t_k)$, $a_i > 0$ ($i = 1, \cdots, n$) are given such that there exists a symmetric positive definite matrix P such that

$$A^\top P + PA \ \le\ -I, \tag{4}$$

where $A = \begin{bmatrix} -a_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \cdots & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix}$.

Note that $e_1(t_k)$ is a constant on $[t_k + \tau_k, t_k + T + \tau_{k+1})$ for $k \ge 0$ and $f_i(\cdot)$ ($i = 1, \cdots, n$) are continuous and satisfy the condition (2), then, $\lim_{t\to t_k+T+\tau_{k+1}^-} \hat{x}_i(t) = \lim_{t\to t_k+T+\tau_{k+1}^+} \hat{x}_i(t)$ ($i = 1, \cdots, n$). Therefore, $\hat{x}_i(t)$ ($i = 1, \cdots, n$) are continuous on $[t_0, \infty)$.

Remark 1. Note that the state estimate is described in continuous time while the evolution process $x_1(t_k) - \hat{x}_1(t_k)$ is only updated at time instants $t_k + \tau_k$, that is, the current evolution process is used until the new evolution process is available. Therefore, the dynamics of observer (3) is of continuous and of hybrid nature.

Remark 2. Even though τ_k and τ_{k+1} are unknown, $e_1(t_k)$ is updated automatically in the observer whenever sampled and delayed measurement arrives. In (Ahmed-Ali et al [2013b]), the sampled and delayed measurement is available at instant $t_k + \tau_k$, however, $e_1(t_k)$ is updated at instant $t_k + \tau$, that is, there exists time delay $\tau - \tau_k$.

Remark 3. In this paper, the sampling period T and the maximum allowable delay $\bar{\tau}$ depend on the Observer (3).

From (1)-(3), for $k \ge 0$, we obtain the observation error:

$$\begin{cases} \dot{e}_1(t) = e_2(t) - La_1 e_1(t_k) + \tilde{f}_1, \\ \dot{e}_2(t) = e_3(t) - L^2 a_2 e_1(t_k) + \tilde{f}_2, \\ \quad \vdots \\ \dot{e}_{n-1}(t) = e_n(t) - L^{n-1} a_{n-1} e_1(t_k) + \tilde{f}_{n-1}, \\ \dot{e}_n(t) = -L^n a_n e_1(t_k) + \tilde{f}_n, \\ \qquad t \in [t_k + \tau_k, t_k + T + \tau_{k+1}), \end{cases} \tag{5}$$

where $e_i(t) = x_i(t) - \hat{x}_i(t)$, $\tilde{f}_i = f_i(x_1(t), x_2(t), \cdots, x_i(t)) - f_i(\hat{x}_1(t), \hat{x}_2(t), \cdots, \hat{x}_i(t))$, $1 \le i \le n$.

Consider the following change of coordinates

$$\varepsilon_i(t) = \frac{e_i(t)}{L^{i-1+b}},\ i = 1, 2, \cdots, n, \tag{6}$$

where b is a positive real number. Then, we have

$$\begin{cases} \dot{\varepsilon}_1(t) = L\varepsilon_2(t) - La_1\varepsilon_1(t_k) + \dfrac{\tilde{f}_1}{L^b}, \\ \dot{\varepsilon}_2(t) = L\varepsilon_3(t) - La_2\varepsilon_1(t_k) + \dfrac{\tilde{f}_2}{L^{1+b}}, \\ \quad \vdots \\ \dot{\varepsilon}_{n-1}(t) = L\varepsilon_n(t) - La_{n-1}\varepsilon_1(t_k) + \dfrac{\tilde{f}_{n-1}}{L^{n-2+b}}, \\ \dot{\varepsilon}_n(t) = -La_n\varepsilon_1(t_k) + \dfrac{\tilde{f}_n}{L^{n-1+b}}, \\ \qquad t \in [t_k + \tau_k,\ t_k + T + \tau_{k+1}). \end{cases} \tag{7}$$

Further, (7) can be rewritten as follows:

$$\begin{cases} \dot{\varepsilon}_1(t) = L\varepsilon_2(t) - La_1\varepsilon_1(t) + La_1(\varepsilon_1(t) - \varepsilon_1(t_k)) \\ \qquad + \dfrac{\tilde{f}_1}{L^b}, \\ \dot{\varepsilon}_2(t) = L\varepsilon_3(t) - La_2\varepsilon_1(t) + La_2(\varepsilon_1(t) - \varepsilon_1(t_k)) \\ \qquad + \dfrac{\tilde{f}_2}{L^{1+b}}, \\ \quad \vdots \\ \dot{\varepsilon}_{n-1}(t) = L\varepsilon_n(t) - La_{n-1}\varepsilon_1(t) + La_{n-1}(\varepsilon_1(t) \\ \qquad - \varepsilon_1(t_k)) + \dfrac{\tilde{f}_{n-1}}{L^{n-2+b}}, \\ \dot{\varepsilon}_n(t) = -La_n\varepsilon_1(t) + La_n(\varepsilon_1(t) - \varepsilon_1(t_k)) + \dfrac{\tilde{f}_n}{L^{n-1+b}}, \\ \qquad t \in [t_k + \tau_k,\ t_k + T + \tau_{k+1}). \end{cases} \tag{8}$$

Now, we will give our main results.

Theorem 1. Consider the system (1) with the condition (2). The output $y(t)$ is assumed to be sampled at instants t_k and is available for the observer at instants