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Adaptive Leader-following Consensus of Multiple Uncertain Rigid Spacecraft Systems ⋆

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Abstract: In this paper, we consider the leader-following consensus problem for multiple uncertain rigid spacecraft systems with the attitude being represented by unit quaternion. Existing results on this problem rely on the assumption that all parameters of the rigid spacecraft system are known exactly. By employing a nonlinear distributed observer for the leader system, we first convert the leader-following consensus problem into a global adaptive stabilization problem of a well defined error system. Then, under the standard assumption that the state of the leader can reach every follower through a path, we further show that this stabilization problem is solvable by a distributed adaptive control law.

Keywords: Multi-agent system, attitude consensus, nonlinear distributed observer, adaptive control.

1. INTRODUCTION

One of the key issues in formation flying of a group of spacecraft systems is to asymptotically aline the attitude and angular velocity of all spacecraft systems to the desired attitude and angular velocity generated by a reference system called leader system. Such a problem is also called leader-following consensus of multiple spacecraft systems.

Depending on whether or not the state of the leader is accessible to all followers, there are roughly two control schemes for dealing with the leader-following consensus problem: decentralized control and distributed control. The former one assumes that the state of the leader is available to all the followers (Abdessameud and Tayebi (2009); VanDyke and Hall (2006)), while the latter one only requires the state of the leader can pass to each of the followers through a path (Bai et al. (2008); Cai and Huang (2014); Ren (2007)). When some followers cannot access the state of the leader, the first scheme is not applicable. One can only make use of local information determined by a communication graph to handle the problem, which has to resort to a distributed control law. Recently, some attempts have been made to deal with the second scenario Bai et al. (2008), Cai and Huang (2014), Ren (2007). The results in Bai et al. (2008) and Ren (2007) have both achieved leader-following consensus for angular velocity. However, in Bai et al. (2008), the consensus for attitude has been realized in a leaderless way, i.e., the attitudes of all followers will converge to a common trajectory determined by the initial condition. In Ren (2007), the leader-following attitude consensus is only applicable to some special type of communication topologies such as a tree. More recently, the authors of this paper solved the leader-following consensus problem for multiple rigid spacecraft systems in Cai and Huang (2014) under the same assumption on the communication graph as in Bai et al. (2008). The result in Cai and Huang (2014) has two features. First, a marginally stable linear system is introduced to generate the desired angular velocity. This scheme enables the control law to handle a class of reference trajectories including step signal with arbitrary magnitude, sinusoidal signal with arbitrary amplitude and initial phase and the combination of the step signal and the sinusoidal signal. Second, the control law achieves both attitude and angular velocity tracking. It is noted that a key technique developed in Cai and Huang (2014) is a nonlinear distributed observer for the leader system which will also play an important role in this paper.

Like all previous papers on leader-following consensus problem of multiple spacecraft systems, the result in Cai and Huang (2014) assumed that all parameters in the spacecraft system are known precisely. This is a quite unrealistic assumption as the mass distribution of the spacecraft system is practically uncertain and may change with time due to fuel consumption or spacecraft reconfiguration (Ahmed et al. (1998); Luo et al. (2005)). To make the result of Cai and Huang (2014) more practically useful, in this paper, we will further consider the leader-following attitude consensus problem without the exact knowledge of the inertial matrix of the spacecraft system. To this end, by employing the same nonlinear distributed observer for the leader system as the one in Cai and Huang (2014), we first convert the leader-following consensus problem into a global adaptive stabilization problem of a well defined error system. Then, under the

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standard assumption that the state of the leader can reach every follower through a path, we further show that this stabilization problem is solvable by a distributed adaptive control law. Finally, the effectiveness of our control scheme is evaluated by simulation.

For the rest of this paper, we use the following notation. $\otimes$ denotes the Kronecker product of matrices. 1_N denotes an N dimensional column vector whose components are all 1. $||x||$ denotes the Euclidean norm of vector x and $||A||$ denotes the Euclidean norm of matrix A. For $x_i \in R^{n_i}$, $i = 1, ..., m$, $\mathrm{col}(x_1, \cdots, x_m) = [x_1^T, \cdots, x_m^T]^T$. For $x = \mathrm{col}(x_1, x_2, x_3) \in R^3$, define

$$x^\times = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.$$

It can be verified that $x^T x^\times = 0$.

2. PROBLEM FORMULATION

We consider a group of N rigid spacecraft systems with the following motion equations:

$$\dot{\hat{q}}_i = \frac{1}{2}\hat{q}_i^\times \omega_i + \frac{1}{2}\bar{q}_i\omega_i, \quad \dot{\bar{q}}_i = -\frac{1}{2}\hat{q}_i^T \omega_i \tag{1a}$$

$$J_i\dot{\omega}_i = -\omega_i^\times J_i\omega_i + u_i, \; i = 1, \ldots, N \tag{1b}$$

where $q_i = \mathrm{col}(\hat{q}_i, \bar{q}_i)$ with $\hat{q}_i \in R^3$, $\bar{q}_i \in R$ is the unit quaternion expression of the attitude of the body frame $\mathcal{B}_i$ of the i^{th} spacecraft relative to the inertial frame $\mathcal{I}$. $\omega_i \in R^3$ is the angular velocity of $\mathcal{B}_i$ relative to $\mathcal{I}$; $J_i \in R^{3\times3}$ is the positive definite inertia matrix; $u_i \in R^3$ is the control torque. ω_i, J_i and u_i are all expressed in $\mathcal{B}_i$.

Following the notation introduced as follows (Tayebi (2008)), for two quaternion $q_i = \mathrm{col}(\hat{q}_i, \bar{q}_i)$ and $q_j = \mathrm{col}(\hat{q}_j, \bar{q}_j)$ with $\hat{q}_i, \hat{q}_j \in R^3$, $\bar{q}_i, \bar{q}_j \in R$, the product of q_i and q_j is given by

$$q_i \odot q_j = \begin{pmatrix} \bar{q}_i\hat{q}_j + \bar{q}_j\hat{q}_i + \hat{q}_i^\times \hat{q}_j \\ \bar{q}_i\bar{q}_j - \hat{q}_i^T \hat{q}_j \end{pmatrix}$$

and the conjugate of q_i is given by $q_i^* = \mathrm{col}(-\hat{q}_i, \bar{q}_i)$. If q_i is a unit quaternion, then its inverse is given by $q_i^{-1} = q_i^*$.

As in Chen and Huang (2009), we assume that the desired attitude q_0 of system (1) is generated by the following system

$$\dot{\hat{q}}_0 = \frac{1}{2}\hat{q}_0^\times \omega_0 + \frac{1}{2}\bar{q}_0\omega_0, \quad \dot{\bar{q}}_0 = -\frac{1}{2}\hat{q}_0^T \omega_0 \tag{2}$$

where $q_0 = \mathrm{col}(\hat{q}_0, \bar{q}_0)$ with $\hat{q}_0 \in R^3$, $\bar{q}_0 \in R$ represents the attitude of the leader frame $\mathcal{B}_0$ relative to the inertial frame $\mathcal{I}$; $\omega_0 \in R^3$ is the angular velocity of $\mathcal{B}_0$ relative to $\mathcal{I}$, expressed in $\mathcal{B}_0$.

Like in Cai and Huang (2014), we view the system composed of (1) and (2) as a multi-agent system of $(N+1)$ agents with (2) as the leader and the N subsystems of (1) as N followers. Given (1) and (2), we can define a graph [1] $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ with $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. Here the node 0 is associated with the leader system (2) and the node i, $i = 1, \ldots, N$, is associated with the i^{th} subsystem of the follower system (1). For $i = 0, 1, \ldots, N$, $j = 1, \ldots, N$, $(i, j) \in \bar{\mathcal{E}}$ if and only if u_j can use the full state of agent i for control. Let $\bar{\mathcal{N}}_i$ denote the neighbor set of the node i of $\bar{\mathcal{G}}$. We can further define a subgraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of $\bar{\mathcal{G}}$ where $\mathcal{V} = \{1, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is obtained from $\bar{\mathcal{E}}$ by removing all the edges between node 0 and the nodes in $\mathcal{V}$.

[1] See Appendix for a summary of graph.

In terms of $\bar{\mathcal{G}}$, we can describe a distributed control law as follows, for $i = 1, \ldots, N$,

$$u_i = k_i(q_i, \omega_i, \varphi_i, \psi_i) \tag{3a}$$

$$\dot{\psi}_i = f_i(q_i, \omega_i, \varphi_i) \tag{3b}$$

$$\dot{\varphi}_i = g_i(\varphi_i, \varphi_j - \varphi_i, j \in \bar{\mathcal{N}}_i) \tag{3c}$$

where k_i, f_i and g_i are smooth functions, and $\varphi_0 = \mathrm{col}(q_0, \omega_0)$.

We introduce the attitude and angular velocity errors between systems (1) and (2) as follows:

$$\epsilon_i = q_0^{-1} \odot q_i, \tag{4a}$$

$$\hat{\omega}_i = \omega_i - C_i\omega_0 \tag{4b}$$

where $\epsilon_i = \mathrm{col}(\hat{\epsilon}_i, \bar{\epsilon}_i)$, $\hat{\epsilon}_i \in R^3$, $\bar{\epsilon}_i \in R$ and $C_i = (\bar{\epsilon}_i^2 - \hat{\epsilon}_i^T\hat{\epsilon}_i)I_3 + 2\hat{\epsilon}_i\hat{\epsilon}_i^T - 2\bar{\epsilon}_i\hat{\epsilon}_i^\times$ is called the direction cosine matrix, which represents the relative attitude between $\mathcal{B}_i$ and $\mathcal{B}_0$. Then, we have

$$\dot{\hat{\epsilon}}_i = \frac{1}{2}\hat{\epsilon}_i^\times \hat{\omega}_i + \frac{1}{2}\bar{\epsilon}_i\hat{\omega}_i, \quad \dot{\bar{\epsilon}}_i = -\frac{1}{2}\hat{\epsilon}_i^T \hat{\omega}_i \tag{5a}$$

$$J_i\dot{\hat{\omega}}_i = -\omega_i^\times J_i\omega_i + J_i(\hat{\omega}_i^\times C_i\omega_0 - C_i\dot{\omega}_0) + u_i. \tag{5b}$$

Remark 1. Note that for $i = 0, 1, \ldots, N$, $||q_i(0)|| = 1$ implies $||q_i(t)|| = 1$ for all $t \geq 0$. Also, by Proposition 1 of Yuan (1988), $\mathcal{B}_i$ and $\mathcal{B}_0$ coincide if and only if $\hat{\epsilon}_i = 0$.

We now state our problem as follows.

Problem 1. Given systems (1), (2) and the graph $\bar{\mathcal{G}}$, design a control law of the form (3) such that, for $i = 1, \ldots, N$,

$$\lim_{t\to\infty} \hat{\epsilon}_i(t) = 0 \text{ and } \lim_{t\to\infty} \hat{\omega}_i(t) = 0$$

for all $\omega_i(0)$ and all $q_i(0)$ satisfying $||q_i(0)|| = 1$.

Remark 2. When $N = 1$, the above problem reduces to the problem studied in Ahmed et al. (1998). If every follower can receive the state from the leader, the approach in Ahmed et al. (1998) can be directly extended to handle Problem 1 and this kind of control scheme is called decentralized control. What makes our current problem interesting is that, without the assumption that every follower can receive the state from the leader, we can still solve the leader-following attitude consensus problem by a distributed control law of the form (3).

We need the following assumptions.

Assumption 1. $\bar{\mathcal{G}}$ contains a spanning tree with the node 0 as the root.

Assumption 2. The desired angular velocity ω_0 is generated by the following system

$$\dot{\omega}_0 = S\omega_0 \tag{6}$$

which is marginally stable with constant matrix $S \in R^{3\times3}$.

Remark 3. Assumption 1 is the standard assumption in consensus problem that imposes a mild constraint on the information exchange among agents. Assumption 2 is made so that the desired angular velocity ω_0 can be estimated by every follower. System (6) can generate step functions of arbitrary magnitudes and sinusoidal functions of arbitrary amplitudes and initial phases.