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Optimal Tube Following for Robotic Manipulators ⋆

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Abstract: Optimal *path following* for robots considers the problem of moving along a predetermined Cartesian geometric end effector path (which is transformed into a predetermined geometric joint path), while some objective is minimized: e.g. motion time or energy loss. In practice it is often not required to follow a path exactly but only within a certain tolerance. By deviating from the path, within the allowable tolerance, one could gain in optimality. In this paper, we define the allowable deviation from the path as a tube around the given geometric path. We then search for the optimal motion inside the tube. This transforms the path following problem to a *tube following* problem. In contrast to the (time or energy) optimal path following problem, the tube following problem is not convex. However, we propose a problem formulation that can still be solved efficiently, as will be illustrated by some numerical examples.

1. INTRODUCTION

Robot path following problems determine the motion of a robot along a predetermined geometric Cartesian end effector path without any preassigned timing information. Common practice is to transform the Cartesian path into a joint path using the inverse kinematics. Path following is often considered to be the low level stage in a decoupled motion planning approach (Bobrow et al., 1985; Shin and Mckay, 1985; Van Loock et al., 2013a), since the motion planning problem (path planning and following) is difficult and highly complex to solve in its entirety (von Stryk and Bulirsch, 1992; Diehl et al., 2005). First, a high level path planner determines a geometric path, ignoring the system dynamics but taking into account geometric path constraints. Second, an optimal trajectory along the geometric path is determined that takes the system dynamics and limitations into account. Since the dynamics along a geometric path can be described by a scalar path coordinate s and its time derivatives (Bobrow et al., 1985; Shin and Mckay, 1985; Van Loock et al., 2013a), the decoupled approach simplifies the motion planning problem to great extent. Furthermore, the path following problem in joint space for a robotic manipulator with simplified constraints can be cast as a *convex* optimization problem (Verscheure et al., 2009a,b). This guarantees efficient computation of globally optimal solutions.

In many applications, the Cartesian geometric end effector path planned by the path planner does not need to be followed exactly but within certain position and orientation tolerances. Typical examples are milling robots where some geometrical tolerance on the workpiece is given. By deviating from the predetermined path, within the allowable tolerance one could gain in optimality.

In (Van Loock et al., 2013b) an optimal path following formulation is presented that provides freedom on the joint path. This freedom on the joint path then results in freedom on the Cartesian path. However, for the applications considered in this paper, where the tolerances are specified in Cartesian space, this formulation should be extended because Cartesian tolerances cannot be transformed back into joint tolerances for robotic manipulators in general.

This paper presents a method that combines freedom on the joint paths, as proposed in (Van Loock et al., 2013b), with constraints on the end-effector Cartesian position and orientation which correspond to the given end effector tolerances. The Cartesian position constraints translate into a tube around the given geometric Cartesian path, to which the end effector is bounded. The freedom on the joint paths is taken sufficiently large such that it is not the restricting factor in the optimization.

The resulting *tube following* problem is nonconvex. In this paper we propose a problem formulation, starting from the path following formulation, that can still be solved efficiently using a standard interior point solver.

This paper is organised as follows. Section 2 reviews the path following problem formulation given in (Verscheure et al., 2009a). Then, Section 3 extends this path following problem to a tube following problem. Here we review the joint path parametrisation given in (Van Loock et al., 2013b) and we define tube constraints on the end effector position and constraints on the orientation of the end effector. Section 4 illustrates the proposed framework with some numerical examples of time-optimal and energy-optimal tube following respectively.

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Throughout the paper we will use the following shorthand notations for the derivatives of a function $f(s(t))$: $\dot{f} = \frac{df}{dt}, \ddot{f} = \frac{d^2 f}{dt^2}, f' = \frac{\partial f}{\partial s}, f'' = \frac{\partial^2 s}{\partial s^2}$ where t indicates time and s the path coordinate. Furthermore, we indicate scalars with a lower-case letter, e.g. n, vectors with a bold lower-case letter, e.g. $\boldsymbol{q}$, and matrices with an upper-case letter, e.g. M. $\boldsymbol{q}_i$ denotes the i-th element of $\boldsymbol{q}$.

2. OPTIMAL PATH FOLLOWING PROBLEM FORMULATION

Consider a robotic manipulator with n degrees of freedom and joint angles $\boldsymbol{q} \in \mathbb{R}^n$. The equations of motion are given by

$$\boldsymbol{\tau} = M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\psi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}), \qquad (1)$$

where $\boldsymbol{\tau} \in \mathbb{R}^n$ are the joint torques, $M \in \mathbb{R}^{n\times n}$ is the mass matrix, $C \in \mathbb{R}^{n\times n}$ is a matrix, linear in $\dot{\boldsymbol{q}}$, accounting for Coriolis and centrifugal effects and, $\boldsymbol{g}$ is a vector accounting for gravity and other position dependent torques.

Consider a prescribed geometric path $\boldsymbol{q}(s)$ as a function of a scalar path coordinate s, given in joint space coordinates. The time dependence of the path is determined through $s(t)$. Without loss of generality it is assumed that the trajectory starts at $t = 0$, ends at $t = T$ and, $0 = s(0) \leq s(t) \leq s(T) = 1$. It is furthermore assumed that we always move forward along the path, i.e. $\dot{s}(t) \geq 0, \forall t \in [0, T]$.

Using the chain-rule we rewrite joint velocities and accelerations as

$$\dot{\boldsymbol{q}}(s) = \boldsymbol{q}'(s)\dot{s} \ \text{ and, } \ \ddot{\boldsymbol{q}}(s) = \boldsymbol{q}''(s)\dot{s}^2 + \boldsymbol{q}'(s)\ddot{s}.$$

Substitution of the above equations in (1) projects the equations of motion onto the path (Verscheure et al., 2009a):

$$\boldsymbol{\tau}(s(t)) = \boldsymbol{\psi}_s\left(s(t), \dot{s}(t)^2, \ddot{s}(t), \boldsymbol{q}(s(t)), \boldsymbol{q}'(s(t)), \boldsymbol{q}''(s(t))\right).$$

Now, by using the same transformation of variables as in (Verscheure et al., 2009a; Van Loock et al., 2013a) we transform the problem from a time t dependent problem into a path s dependent problem where we use s as an independent variable instead of time t.

$$\dot{s}^2 = b(s), \text{ where } \ddot{s} = \frac{1}{2}b'(s).$$

This results in the following dynamics

$$\boldsymbol{\tau}(s) = \boldsymbol{\psi}_b\left(s, b(s), b'(s), \boldsymbol{q}(s), \boldsymbol{q}'(s), \boldsymbol{q}''(s)\right).$$

2.1 Time-optimal path following

The total motion time is given by

$$T = \int_0^T 1 dt = \int_0^1 \frac{1}{\dot{s}} ds = \int_0^1 \frac{1}{\sqrt{b(s)}} ds.$$

The time-optimal path following problem is then formulated as

$$\begin{aligned} \underset{b(\cdot),\boldsymbol{\tau}(\cdot)}{\text{minimize}} \quad & \int_0^1 \frac{1}{\sqrt{b(s)}} ds \\ \text{subject to} \quad & b(0) = \dot{s}_0^2, b(1) = \dot{s}_T^2, b(s) \geq 0 \\ & \boldsymbol{\tau}(s) = \boldsymbol{\psi}_b\left(s, b(s), b'(s), \boldsymbol{q}(s), \boldsymbol{q}'(s), \boldsymbol{q}''(s)\right) \\ & \boldsymbol{\tau}_- \leq \boldsymbol{\tau}(s) \leq \boldsymbol{\tau}_+ \\ & \text{for } s \in [0, 1]. \end{aligned} \qquad (2)$$

Once the optimal solution for $b(\cdot), \boldsymbol{\tau}(\cdot)$ is obtained, the relation between path coordinate and time can be obtained from the relation

$$t(s) = \int_0^s \frac{1}{\sqrt{b(\sigma)}} d\sigma.$$

Note that optimization problem (2) is a fixed end-time problem due to the transformation from time domain t to path domain s. In general this is much easier to solve than a free end-time problem due to the strongly non-linear dependence of the solution with varying end-times.

This time-optimal path following problem (hence fixed $\boldsymbol{q}(s)$) is *convex* for a simplified robot and simple task constraints (Verscheure et al., 2009a; Debrouwere et al., 2012). In the extension to tube following we will allow deviations from the fixed joint path, hence $\boldsymbol{q}(s)$ is free as in (Van Loock et al., 2013b) and the optimization problem is *nonconvex*. The proposed problem formulation, given in the following section, results in an numerical optimization problem which can be solved efficiently using standard nonconvex solvers.

2.2 Energy-optimal path following

In (Verscheure et al., 2009a) a trade-off is made between thermal energy losses and motion time. Hence energy loss minimization results in larger motion times. The thermal losses are dominated by the electrical resistive energy loss for each joint i which is proportional to integral of the square of the joint motor torque. The total thermal losses are then proportional to:

$$\sum_{i=1}^{n} \int_0^T \frac{\boldsymbol{\tau}_i^2}{\boldsymbol{\tau}_{+,i}^2} dt = \sum_{i=1}^{n} \int_0^1 \frac{\boldsymbol{\tau}_i^2}{\boldsymbol{\tau}_{+,i}^2 \sqrt{b(s)}} ds, \qquad (3)$$

where $\boldsymbol{\tau}_{+,i}^2$ is used as a normalization factor.

It can be shown (Boyd and Vandenberghe, 2004) that $x^2/\sqrt{y}$ is a convex function of (x, y), for $y \geq 0$, hence the electrical energy losses are convex.

Another approach could be to only minimize the energy losses (3) while constraining the motion time to some maximal value T_m. The energy-optimal path following problem is similar to the time-optimal path following problem (2), however it has a different objective function (3), and an additional constraint $\int_0^1 b(s)^{-1/2} ds \leq T_m$. This energy-optimal approach extends easily to tube following. The extra freedom (deviation from the Cartesian path within the tube) allows the robot to deviate from the nominal path to minimize the energy losses while preserving the motion time T_0^* of the time-optimal path following problem. Hence $T_m = T_0^*$.

The following section extends the time-optimal path following problem to time-optimal tube following problems. The energy-optimal tube following problem derivation is straightforward.

3. FROM PATH FOLLOWING TO TUBE FOLLOWING

Generally a robot task is specified in Cartesian coordinates of the end effector pose $\boldsymbol{y}(s) = (x, y, z, \phi, \theta, \psi)^T =$