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A proportional integral extremum-seeking control approach

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Abstract: This paper proposes an alternative extremum seeking control design technique for the solution of real-time optimization control problems. The technique considers a proportional-integral approach that avoids the need for a time-scale separation in the formulation of the ESC. It is assumed that the equations describing the dynamics of the nonlinear system and the cost function to be minimized are unknown and that the objective function is measured. The dynamics are assumed to be asymptotically stable and relative order one with respect to the objective function. The extremum-seeking problem is solved using a time-varying parameter estimation technique.

Keywords: Extremum-seeking control, Real-time optimization, Time-varying systems

1. INTRODUCTION

Extremum-seeking control (ESC) has been the subject of considerable research effort over the last decade. This approach, which dates back to the 1920s Leblanc [1922], is an ingenious mechanism by which a system can be driven to the optimum of a measured variable of interest Tan et al. [2010]. The revived interest in the field was primarily sparked by Krstic and co-workers who provided an elegant proof of the convergence of a standard perturbation based extremum seeking scheme for a general class of nonlinear systems. The main drawback of ESC is the lack of transient performance guarantees. As highlighted in the proof of Krstic and Wang Krstic and Wang [2000], the stability analysis relies on two components: 1) an averaging analysis of the persistently perturbed ESC loop and 2) a time-scale separation of ESC closed-loop dynamics between the fast transients of the system dynamics and the slow quasi steady-state extremum-seeking task.

Over the last few years, many researchers have considered various approaches to overcome the limitations of ESC. In Krstic [2000], the performance limitations associated with ESC were considered in detail. The non-local properties on ESC was studied in Tan et al. [2006]. This work extends the work in Krstic and Wang [2000] by considering the case where the fast dynamics can be assumed to be uniformly global asymptotically stable along the equilibrium manifold. In Adetola and Guay [2007], Guay et al. [2004] and Cougnon et al. [2011], an alternative ESC algorithm is considered where an adaptive control and estimation approach is used. The key aspect of this approach is that the equilibrium map is parameterized and the parameters are estimated with the help of a tailored adaptive estimation technique. The results in Nesic et al. [2010] unify the approaches based on singular perturbation and parameter estimation by considering the case where the objective function is parameterized in a known fashion. A three-time scale approach is proposed to establish the combined adaptive estimation and extremum seeking control algorithms. Recent work reported in Ghaffari et al. [2012] and Moase et al. [2010] have proposed a Newton-based extremum-seeking technique that provides an estimate of the inverse of the Hessian of the cost function. This technique can effectively alleviate the convergence problems associated with the increase of the gain of the Newton update. Other alternative techniques such as proposed Zhang and Ordóñez [2009] and Zhang and Ordóñez [2012] make use of sampled gradient measurements to improve the convergence properties of ESC techniques that implement numerical optimization techniques. A sliding-mode approach is presented in Fu and Özgüner [2011].

Although the limitations associated with the tuning of ESC is generally well understood, the limitations associated with the two time-scale approach to ESC remains problematic. Under the two time-scale assumption, the optimization operates at a quasi steady-state, or slow, time-scale such that the search for optimal operating conditions does not affect the process dynamics. To overcome the time-scale separation, one must incorporate some knowledge of the transient behaviour of the process dynamics. In the case where a model is available, one can use adaptive extremum seeking technique as proposed in Guay and Zhang [2003] to stabilize a nonlinear system to the unknown optimum of a known but unmeasured cost function. If a model is not available but similar systems are available, the use of multi-unit extremum seeking control techniques Srinivasan [2007] can be used to steer both systems in a neighbourhood of the unknown optimum. Both classes of techniques can solve the steady-state optimization ESC problem without the need for time-scale separation. In Scheinker and Krstic [2013], Lie bracket averaging techniques are considered to stabilize unknown dynamical systems using ESC. The approach does not explicitly rely on the need for time-scale separation but it requires a known CLF of the unknown control system.

ESC problems cannot be currently solved in the absence of time-scale separations if explicit process models or multiple identical units are not available. This paper attempts to bridge this gap in the application of ESC. It proposes a proportional-integral ESC design technique. This technique can be interpreted as a generalization of the standard approach where the integral action corresponds to the standard ESC control task used to identify the steady-state optimum. The proportional control action is designed to ensure that the measured cost function is optimized instantaneously. The approach considers an alternative parameterization of the ESC problem in which the rate of change of the output is parameterized directly without the need to invoke a time-scale separation argument. Under suitable assumption on the dynamics of the system, this action can be shown to minimize the cost over short times while reaching the optimum steady-state conditions.

The paper is organized as follows. A brief description of the ESC problem is given in section 2. In section 3, the proposed ESC formulation is presented for a known cost function and process dynamics. The proposed proportional-integral ESC controller is described in section 4. A simulation example is presented in 5 followed by brief conclusions in 6.

2. PROBLEM DESCRIPTION

Consider a nonlinear system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x) \tag{2}$$

where $x \in \mathbb{R}^n$ is the vector of state variables, u is the vector of input variables taking values in $\mathcal{U} \subset \mathbb{R}^p$ and $y \in \mathbb{R}$ is the variable to be minimized. It is assumed that $f(x)$ and $g(x)$ a smooth vector valued functions of x and that $h(x)$ is a smooth function of x.

The objective is to steer the system to the equilibrium x^* and u^* that achieves the minimum value of $y(= h(x^*))$. The equilibrium (or steady-state) map is the n dimensional vector $\pi(u)$ which is such that:

$$f(\pi(u)) + g(\pi(u))u = 0.$$

The equilibrium cost function is given by:

$$y = h(\pi(u)) = \ell(u) \tag{3}$$

Thus, at equilibrium, the problem is reduced to finding the minimizer u^* of $y = \ell(u^*)$. Let $\mathcal{D}(u)$ be a neighbourhood of the steady-state $x = \pi(u)$.

Some additional assumptions are required concerning the cost function $h(x)$.

Assumption 1. The cost $h(x)$ is such that

(1) $\frac{\partial h(x^*)}{\partial x} = 0$
(2) $\frac{\partial^2 h(x)}{\partial x \partial x^T} > \alpha I, \ \forall x \in \mathbb{R}^n$

where α is a strictly positive constant.

Note that, in contrast to standard ESC, convexity of the cost function $h(x)$ is required. We also require the following properties for the dynamics:

Assumption 2. The dynamics (1) are such that:

(1) the cost function $h(x)$ decreases in the direction of $f(x)$:
$$\frac{\partial h}{\partial x} f(x) + \frac{\partial h}{\partial x} g(x)u \le -\alpha \|x - \pi(u)\|^2, \ \forall x \in \mathcal{D}(u),$$
(2) the matrix valued function $g(x)$ is full rank $\forall x \in \mathcal{D}(u)$,

$\forall u \in \mathcal{U}$.

Assumption 2 states that h is non-decreasing in along the vector field $f(x) + g(x)u$ over some neighbourhood of the steady-state manifold $x = \pi(u)$ at a fixed value of the input u. It also states that the cost function is relative order 1 in a neighbourhood of the origin.

Finally, we will require the following additional assumption concerning the steady-state cost function $\ell(u)$.

Assumption 3. The equilibrium steady-state map $\ell(u)$ is such that

$$\nabla_u \ell(u)(u - u^*) \ge \alpha_u \|u - u^*\|^2$$

for some positive constant α_u $\forall u \in \mathcal{U}$.

3. EXTREMUM SEEKING CONTROLLER WITH FULL INFORMATION

In this section, we propose the extremum-seeking control approach that will form the basis of the development in later sections. Let us first consider the cost function $y = h(x)$ and compute its time derivation:

$$\dot{y} = L_f h + L_g h u \tag{4}$$

where $L_f h$ and $L_g h$ are the Lie derivatives of $h(x)$ with respect to $f(x)$ and $g(x)$, respectively. The Lie derivative is the directional derivative of the function $h(x)$ given by:

$$L_f h = \frac{\partial h}{\partial x} f, \ L_g h = \frac{\partial h}{\partial x} g.$$

By the relative order assumption it follows that $L_g h \neq 0$ in a neighbourhood of the unknown optimum x^*.

We propose the following controller:

$$u = -k L_g h + \hat{u} \tag{5}$$

where $\hat{u}$ is a steady-state bias term to be estimated. Let the optimal steady-state input be given by u^*. The error in the deviation bias is denoted by $\tilde{u} = u^* - \hat{u}$. Pose the function

$$V = y + \frac{1}{2} \tilde{u}^T \tilde{u}$$

Its time derivative is given by:

$$\dot{V} = L_f h - k \|L_f g\|^2 + L_g h \hat{u} - \tilde{u} \dot{\hat{u}}.$$

Let $\dot{\hat{u}} = -L_g h$. Upon substitution of $\tilde{u} = u^* - \hat{u}$, one obtains:

$$\dot{V} = L_f h - k \|L_f g\|^2 + L_g h u^*$$

By assumption, it follows that:

$$\dot{V} \le -\alpha \|x - \pi(u^*)\|^2 - k \|L_g h\|^2$$

Since $g(x)$ is everywhere full rank and x^* is the unique point where $\nabla_x h(x^*) = 0$. Thus the system reaches the