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Real-time simultaneous identification of structural systems and unknown inputs without collocated acceleration measurements based on MEKF-UI

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ABSTRACT

Identifications of excitations and structural systems are two crucial factors in structural health monitoring. There have been some research works focusing on simultaneous identification of structural systems and unknown inputs. Among them, the approach based on the extended Kalman filter with unknown inputs (EKF-UI) has the superiority of implementing identification with limited measurements and in real-time fashion. However, in previous EKF-UI approaches, it is necessary to have the measurements of acceleration responses at the locations where unknown inputs applied. In this paper, it is proposed to circumvent the above limitations for real time simultaneous identification of structural systems and unknown inputs without using collocated acceleration measurements. The proposed method is based on a novel modal extended Kalman filter with unknown inputs (MEFK-UI), which is the combination of extended Kalman filter in modal domain and the recent EKF-UI developed by the authors. Simultaneous identification of structural modal co-ordinates and unknown modal inputs are first conducted in the reduced dimension by modal truncation technique. Moreover, data fusion of displacement and acceleration measurements is used to prevent in real time the low-frequency drifts in the identification results. Subsequently, structural systems including structural parameters and unknown external inputs are identified in real time. Such an approach is not available in the literature. Several numerical examples are used to demonstrate the effectiveness of the proposed method.

1. Introduction

Loads and structural property are two crucial factors in structural health monitoring (SHM) [1–4]. However, it is difficult or even impossible to measure all structural external excitations under actual operating conditions, it is essential to investigate algorithms for the identification of structures as well as the unknown external excitations. However, dealing with unmeasured input has made the identification of the structural parameters a much more challenging task, with the analysis relying only on the information contained in the structural response (output) measurements. In the past, input force identification has been carried out under the assumption that the system is 'completely known', for example, all the system's parameters are known [5–7]. Yet, most of the system's parameters cannot be assumed known a priori (e.g., degradation and deterioration might exist in a structure) but need to be identified. In the last decade, some approaches have been proposed for simultaneous identification of structural systems and unknown external excitations, e.g., the iterative least-squares approaches with unknown excitations [8,9], a hybrid identification method for multi-story buildings with unknown ground motion [10], a quadratic sum-squares error with unknown inputs [11], a weighted adaptive iterative least-squares estimation with incomplete measured excitations (WAILSE-IME) [12], dynamic response sensitivity method [13,14], optimization problem in which structural excitation time history is decomposed by orthogonal approximation [15,16], a virtual distortion method (VDM) [17], and a hybrid heuristic optimization strategy [18].

Since it is impractical to deploy so many sensors to the measure all structural responses [19,20], simultaneous identification of structural systems and unknown inputs using only partial measurements of structural responses is an important but challenging task. In this regard, the approaches based on the extended Kalman filter (EKF) with consideration of unknown input have received great attention [21,22]. Also, based on the recursive estimation by EKF, it is suitable for real-time identification. The authors recently extended the conventional extended Kalman filter (EKF)

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approach to extended Kalman filter with unknown inputs (EKF-UI) [23]. Based on the procedures of the conventional EKF, analytical recursive solutions for the EKF-UI are derived and presented, so it is straightforward, intuitive and easy to be implemented. Moreover, data fusion of displacement and acceleration measurements is used to prevent in real time the low-frequency drifts in the identification results. Such an analytical recursive solution for data fusion based EKF-UI is not available in the previous literature. Several numerical examples have demonstrated the effectiveness and versatilities of the proposed EKF-UI. However, like other previous approaches based on extended Kalman filter with unknown input, it is necessary to have the measurements of acceleration responses at the locations where unknown inputs applied, i.e., collocated acceleration measurements at the locations of unknown inputs are requested.

In this paper, it is proposed to circumvent the above limitations for real time simultaneous identification of structural systems and unknown inputs without using collocated acceleration measurements. The proposed method is based on a novel modal extended Kalman filter with unknown inputs (MEFK-UI), which is the combination of extended Kalman filter in modal domain [24,25] and the recent EKF-UI developed by the authors. Simultaneous identification of structural modal co-ordinates and unknown modal inputs are first conducted in the reduced dimension by modal truncation technique [25,26]. Also, data fusion of displacement and acceleration measurements is used to prevent in real time the low-frequency drifts in the identification results. Subsequently, structural systems including structural parameters and unknown external inputs are identified. Such an approach is not available in the literature. Several numerical examples are used to demonstrate the effectiveness of the proposed method.

The paper is organized as follows: In Section 2, the recent data fusion EKF-UI approach proposed by the authors is briefly reviewed; In Section 3, the proposed MEFK-UI is addressed; In Section 4, two numerical examples are used to demonstrate the performances of proposed MEKF-UI, respectively. Finally, some conclusions are presented.

2. A brief review of the recent data fusion EKF-UI

The extended Kalman filter (EKF) has been widely utilized to identify the structural system and unknown structural dynamics parameters due to its superiority of using only partial measurements of structural dynamic responses. However, the conventional EKF approach is only applicable when the information of external inputs to structures is available. In order to solve above problem, the authors [23] proposed an extended Kalman filter with unknown inputs (EKF-UI) based on the procedures of the conventional EKF.

When some external inputs to the n-DOF structure are unknown, the equation of motion of can be expressed as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{F}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \theta) = \boldsymbol{\eta}\boldsymbol{f} + \boldsymbol{\eta}^u\boldsymbol{f}^u \tag{1}$$

where $\boldsymbol{M}$ is the mass matrix of the structure, $\boldsymbol{x}$, $\dot{\boldsymbol{x}}$ and $\ddot{\boldsymbol{x}}$ are the vectors of the displacement, velocity and acceleration responses, respectively. θ is a l-dimensional unknown structural parametric vector, $\boldsymbol{F}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \theta)$ is the a force vector which can be linear or nonlinear function of the displacements, velocities and the structural parameters, $\boldsymbol{f}$ and $\boldsymbol{\eta}$ denote an known external excitation vector and the influence matrix associated with the excitation $\boldsymbol{f}$, respectively. $\boldsymbol{f}^u$ and $\boldsymbol{\eta}^u$ are a unmeasured p-dimensional external excitation vector and the influence matrix associated with the excitation $\boldsymbol{f}^u$.

By introducing an $2n+l$ dimensional extended state vector $\boldsymbol{Z} = (\boldsymbol{x}^T\dot{\boldsymbol{x}}^T\theta^T)$ and considering modeling error, Eq. (1) can be converted into the following state equation as:

$$\dot{\boldsymbol{Z}} = \boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u) + \boldsymbol{w}(\mathrm{t}) \tag{2}$$

in which, $\boldsymbol{g}(\bullet)$ is a nonlinear function and $\boldsymbol{w}(\mathrm{t})$ is the model noise (uncertainty) with zero mean and a covariance matrix $\mathbf{Q}(\mathrm{t})$.

The nonlinear discrete equation for an observation vector can be expressed as:

$$\boldsymbol{y}_{k+1} = \boldsymbol{h}(\boldsymbol{Z}_{k+1}, \boldsymbol{f}_{k+1}, \boldsymbol{f}^u_{k+1}) + \boldsymbol{v}_{k+1} \tag{3}$$

in which $\boldsymbol{y}_{k+1}$ is a m-dimensional measured acceleration response vector at time $t = (k+1)\Delta t$ with Δt being the sampling time step and $\boldsymbol{v}_{k+1}$ is the measurement noise vector of a Gaussian white noise vector with zero mean and a covariance matrix $E(v_{k+1}v^T_{k+1}) = R_{k+1}$.

Let $\hat{\boldsymbol{Z}}_{k|k}$ and $\hat{\boldsymbol{f}}^u_{k|k}$ be the estimates of $\boldsymbol{Z}_k$ and $\boldsymbol{f}^u_k$ given the observations ($\boldsymbol{y}_1$, $\boldsymbol{y}_2$, ..., $\boldsymbol{y}_k$), respectively, Eq. (2) can be linearized at $\hat{\boldsymbol{Z}}_{k|k}$ and $\hat{\boldsymbol{f}}_{k|k}$ by Taylor series expansion to the first order as:

$$\boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u) \approx \boldsymbol{g}(\hat{\boldsymbol{Z}}_{k|k}, \boldsymbol{f}, \hat{\boldsymbol{f}}^u_{k|k}) + \boldsymbol{G}_{k|k}(\boldsymbol{Z} - \hat{\boldsymbol{Z}}_{k|k}) + \boldsymbol{B}^u_{k|k}(\boldsymbol{f}^u - \hat{\boldsymbol{f}}^u_{k|k}) \tag{4}$$

where

$$\boldsymbol{G}_{k|k} = \left.\frac{\partial \boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{Z}}\right|_{\boldsymbol{Z}=\hat{\boldsymbol{Z}}_{k|k}, \boldsymbol{f}^u=\hat{\boldsymbol{f}}^u_{k|k}}; \quad \boldsymbol{B}^u_{k|k} = \left.\frac{\partial \boldsymbol{g}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{f}^u}\right|_{\boldsymbol{Z}=\hat{\boldsymbol{Z}}_{k|k}, \boldsymbol{f}^u=\hat{\boldsymbol{f}}^u_{k|k}} \tag{5}$$

Analogous to the conventional EKF, the first time update (prediction) procedure is

$$\tilde{\boldsymbol{Z}}_{k+1|k} = \hat{\boldsymbol{Z}}_{k|k} + \int_{k\Delta t}^{(k+1)\Delta t} \boldsymbol{g}(\hat{\boldsymbol{Z}}_{t|k}, \boldsymbol{f}, \hat{\boldsymbol{f}}^u_{k|k})dt \tag{6}$$

The prediction error of $\tilde{\boldsymbol{Z}}_{k+1|k}$ is $\tilde{\boldsymbol{e}}^{\boldsymbol{Z}}_{k+1|k} = \boldsymbol{Z}_{k+1} - \tilde{\boldsymbol{Z}}_{k+1|k} = \boldsymbol{A}^{\boldsymbol{Z}}_{k|k}\hat{\boldsymbol{e}}^{\boldsymbol{Z}}_{k|k} + \Delta t\boldsymbol{B}^u_{k|k}\hat{\boldsymbol{e}}^{\boldsymbol{f}^u}_{k|k} + \boldsymbol{w}_k$, where $\boldsymbol{A}^{\boldsymbol{Z}}_{k|k} \approx \boldsymbol{I}_{2n+l} + \Delta t\boldsymbol{G}_{k|k}$ and $\hat{\boldsymbol{e}}^{\boldsymbol{f}^u}_{k|k}$ is the error of $\hat{\boldsymbol{f}}^u_{k|k}$, i.e., $\hat{\boldsymbol{e}}^{\boldsymbol{f}^u}_{k|k} = \boldsymbol{f}^u_k - \hat{\boldsymbol{f}}^u_{k|k}$.

The observation equation can also be linearized at $\tilde{\boldsymbol{Z}}_{k+1|k}$ and $\hat{\boldsymbol{f}}^u_{k|k}$ by Taylor series expansion to the first order as,

$$\begin{aligned}\boldsymbol{h}(\boldsymbol{Z}_{k+1}, \boldsymbol{f}_{k+1}, \boldsymbol{f}^u_{k+1}) &= \boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}_{k+1}, \hat{\boldsymbol{f}}^u_{k|k}) + \boldsymbol{H}_{k+1|k}(\boldsymbol{Z}_{k+1} - \tilde{\boldsymbol{Z}}_{k+1|k}) \\ &\quad + \boldsymbol{D}^u_{k+1|k}(\boldsymbol{f}^u_{k+1} - \hat{\boldsymbol{f}}^u_{k|k})\end{aligned} \tag{7}$$

where

$$\begin{aligned}\boldsymbol{H}_{k+1|k} &= \left.\frac{\partial \boldsymbol{h}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{Z}}\right|_{\boldsymbol{Z}=\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}^u=\boldsymbol{f}^u_{k|k}}; \quad \boldsymbol{D}^u_{k+1|k} \\ &= \left.\frac{\partial \boldsymbol{h}(\boldsymbol{Z}, \boldsymbol{f}, \boldsymbol{f}^u)}{\partial \boldsymbol{f}^u}\right|_{\boldsymbol{Z}=\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}^u=\boldsymbol{f}^u_{k|k}}\end{aligned} \tag{8}$$

Then, the second measurement update (correction) procedure is

$$\begin{aligned}\hat{\boldsymbol{Z}}_{k+1|k+1} &= \tilde{\boldsymbol{Z}}_{k+1|k} + \boldsymbol{K}_{k+1}[\boldsymbol{y}_{k+1} - \boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}_{k+1}, \hat{\boldsymbol{f}}^u_{k|k}) \\ &\quad - \boldsymbol{D}^u_{k+1|k}(\hat{\boldsymbol{f}}^u_{k+1|k+1} - \hat{\boldsymbol{f}}^u_{k|k})]\end{aligned} \tag{9}$$

where $\hat{\boldsymbol{Z}}_{k+1|k+1}$ and $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ are the estimate of $\boldsymbol{Z}_{k+1}$ and $\boldsymbol{f}^u_{k+1}$ given the observations ($\boldsymbol{y}_1$, $\boldsymbol{y}_2$, ..., $\boldsymbol{y}_{k+1}$), respectively, and $\boldsymbol{K}_{k+1}$ is the Kalman gain matrix, which is derived as,

$$\boldsymbol{K}_{k+1} = \tilde{\boldsymbol{P}}^{\boldsymbol{Z}}_{k+1|k}\boldsymbol{H}^T_{k+1|k}(\boldsymbol{H}_{k+1|k}\tilde{\boldsymbol{P}}^{\boldsymbol{Z}}_{k+1|k}\boldsymbol{H}^T_{k+1|k} + \boldsymbol{R}_{k+1})^{-1} \tag{10}$$

Under the condition that the number of acceleration response measurements is not larger than the total number of unknown external excitations, i.e., m > p, $\hat{\boldsymbol{f}}^u_{k+1|k+1}$ can be estimated by the least-squares estimation as

$$\begin{aligned}\hat{\boldsymbol{f}}^u_{k+1|k+1} &= \boldsymbol{S}_{k+1}\boldsymbol{D}^{uT}_{k+1|k}\boldsymbol{R}^{-1}_{k+1}(\boldsymbol{I}_m - \boldsymbol{H}_{k+1|k}\boldsymbol{K}_{k+1})[\boldsymbol{y}_{k+1} \\ &\quad - \boldsymbol{h}(\tilde{\boldsymbol{Z}}_{k+1|k}, \boldsymbol{f}_{k+1}, \hat{\boldsymbol{f}}^u_{k|k}) + \boldsymbol{D}^u_{k+1|k}\hat{\boldsymbol{f}}^u_{k|k}]\end{aligned} \tag{11}$$

where $\boldsymbol{S}_{k+1} = [\boldsymbol{D}^{uT}_{k+1|k}\boldsymbol{R}^1_{k+1}(\boldsymbol{I}_m - \boldsymbol{H}_{k+1|k}\boldsymbol{K}_{k+1})\boldsymbol{D}^u_{k+1|k}]^1$.