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Extremum Seeking Control With Adaptive Disturbance Feedforward

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Abstract: This paper presents an extension to the classical gradient-based extremum seeking control for the case when the disturbances responsible for changes in the extremum of a selected performance function are available for measurement. Based on these additional measurements, an adaptive extremum seeking disturbance feedforward is designed that approximates the unknown, static mapping between the disturbances and the optimal inputs. For this purpose, orthogonal, multivariate Tchebyshev polynomials are used. The feedforward enables the extremum seeking to be conducted in the proximity of the extremum thus yielding improvements both in terms of accuracy and increased convergence speed compared to the traditional scheme. Simulation results given for a turbine driven electrical generator system demonstrate the benefits of the presented design.

Keywords: Extremum seeking control; Feedforward control; Adaptive algorithms.

1. INTRODUCTION

In a wide variety of control applications the aim is to operate a physical system or a process in the vicinity of an extremum (optimal set-point) of some performance function. Very often the performance function is measurable but unknown to the designer, in terms of its exact analytical dependency on the system parameters (optimizing inputs). In such cases Extremum Seeking Control (ESC) techniques can be used to achieve and maintain the operation of a system under optimal conditions. Numerous reports of successful implementations of ESC can be found in literature, *e.g.,* for improving continuously variable transmission efficiency as in Van der Meulen et al. [2012], or for Maximum Power Point Tracking (MPPT) in photovoltaic (PV), fuel cell and wind energy systems, see Zazo et al. [2012], Bizon [2010] and Pan et al. [2008], respectively.

Extremum Seeking Control was first investigated in the 1950s and 1960s as a control framework for finding a minimum or a maximum value of a static map, see Tan et al. [2006]. However, a rigorous stability proof for the "classical" ESC with a general nonlinear dynamical plant arrived only at the beginning of the past decade, see Krstić and Wang [2000]. Since then there has been a revival of interest and a steady development in the field. Today ESC encompasses various online optimization techniques which can roughly be split into gradient-based as in Krstić and Wang [2000], and gradient-free methods, *e.g.,* sliding mode ESC as in Korovin and Utkin [1974]. However, hybrid algorithms such as the Simplex Guided ESC by Zhang and Gans [2012] – a combination between a local, gradient-based search and a global, gradient-free direct search algorithm, also do exist. Furthermore, one can distinguish between numerical optimization-based, parametric and classical-gradient ESC. The classical gradient-based approach, as in Krstić and Wang [2000], Moura and Chang [2010] and Van de Wouw et al. [2012], is the most popular of all ESC schemes due to its simple implementation and a proof of local convergence. It relies on the fact that a signal proportional to the local gradient of the performance function (w.r.t. to the optimizing input) can be extracted from a product between the sinusoidal input perturbation and the resulting system's response. A simple integration of the gradient estimate (or its negation) is then sufficient to continuously steer the system toward the extremum.

Overall, the classical gradient-based ESC schemes demonstrate good seeking behavior when the extremum is static. However, often the optimal operating point can also change over time. For instance, shifts in solar irradiation and wind speed can cause fluctuations in the optimal PV voltage and the optimal wind turbine rotation speed, see Kumari and Babu [2012] and Munteanu et al. [2009]. To account for such variations in the extremum, Krstić [2000] proposed an extension to the original algorithm by introducing a dynamic compensator into the extremum seeking loop. Still, the solution applies only for the case of changes with known dynamics that can be captured by a linear time-invariant system (*e.g.,* a double integrator).

However, in some practical applications the disturbances leading to changes in the optimal input/extremum value of a selected performance function are measurable. In this paper, we show how this additional information can be used to achieve faster and more accurate convergence of the classical gradient-based ESC scheme. In particular, we use the classical gradient-based ESC both to search for a new extremum and to identify the mapping between the disturbances and the optimal inputs. The mapping is approximated by means of multivariate Tchebychev polynomials whose coefficients are adaptively updated using the latest estimate of the optimal input. Based on the approximate mapping, the proposed solution, *i.e.,* the Adaptive disturbance feedforward ESC (AESC), is able to conduct the search in the close vicinity of the ever-changing extremum. This in turn shortens the convergence times and improves the overall extremum tracking performance. Note that the proposed method is not limited to the classical-gradient ESC but can also be used in combination with other similar ESC algorithms, such as those by Ghaffari et al. [2012], Moase et al. [2010] and Moase and Manzie [2012].

In summary, the main contributions of this paper are as follows. The paper presents a solution to the problem of tracking an unknown, time-varying extremum of a certain performance function. This is achieved by extending the original classical gradient-based ESC scheme with an adaptive disturbance feedforward based on multivariate Tchebychev polynomials. The extension greatly enhances the extremum tracking performance as demonstrated in a turbine driven electrical generator system case study.

This paper is organized as follows. Section II provides the problem formulation, followed by the description of the proposed ESC scheme in Section III. Section IV contains representative simulation results from the case study on a turbine driven electrical generator system. Finally, the main conclusions are given in Section V.

2. PROBLEM FORMULATION

Consider a stable dynamical nonlinear closed-loop system:

$$\dot{x} = f(x, q, \theta), \qquad (1)$$
$$y = h(x),$$

with continuously differentiable $f : \mathbb{R}^n \times \mathbb{R}^l \times \mathbb{R} \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}$. Here $x \in \mathbb{R}^n$ denotes the closed-loop state, $y \in \mathbb{R}$ the *measurable* performance function output, $q \in \mathbb{R}^l$ the *measurable* state disturbance and $\theta \in \mathbb{R}$ denotes a scalar (optimizing) input to the closed loop system.

Let the following assumptions hold:

Assumption 1. The relative degree of (1) w.r.t. the output y and the input θ is at least 1.

Assumption 2. There exists a smooth function $s : \mathbb{R} \times \mathbb{R}^l \to \mathbb{R}^n$ such that $f(x, q, \theta) = 0$, if and only if $x = s(\theta, q)$.

Assumption 3. For each $\theta \in \mathbb{R}$ and $q \in \mathbb{R}^l$, the equilibrium of the system (1), given by $x = s(\theta, q)$, is locally exponentially stable uniformly in θ and q.

Assumption 4. There exists a smooth function $z : \mathbb{R}^l \to \mathbb{R}$ such that for each $q \in \mathbb{R}^l$

$$\frac{\partial h(s(\theta, q))}{\partial \theta}(\theta^*, q) = 0, \qquad (2)$$
$$\frac{\partial^2 h(s(\theta, q))}{\partial \theta^2}(\theta^*, q) = W(q) < 0, \qquad W(q) = W(q)^T,$$

if and only if $\theta^* = z(q)$, see Fig. 1 for illustration. Without loss of generality we thus assume that the extremum of h (w.r.t. θ) is a maximum.

The first assumption removes the possibility of a direct relation between the optimizing input and the performance function output (feedthrough). If omitted, the existence of such a relation would cause the related ESC to optimize it instead of the equilibrium performance of the closed-loop system, which is clearly undesirable. The second relates the equilibria of the system to the input and the disturbance while the third provides guarantees for their stability. Finally, the last assumption requires that there is an equilibrium where the performance function admits a maximal (optimal) value for every value of the disturbance. It also states that the corresponding optimizing input is parameterized by the disturbance. Thus one can proceed with construction of an extremum (optimum) seeking algorithm, with a disturbance feedforward. Note that each of the functions f, s, h and z may be unknown to the designer.

Within this class of systems we treat a problem of finding the input $\theta = \theta^*$ which optimizes the performance function $h(s(\theta, q))$ for each value of the measured disturbance q. In particular, we are interested in the ESC-based solutions yielding an approximation of the unknown mapping $z(q)$.

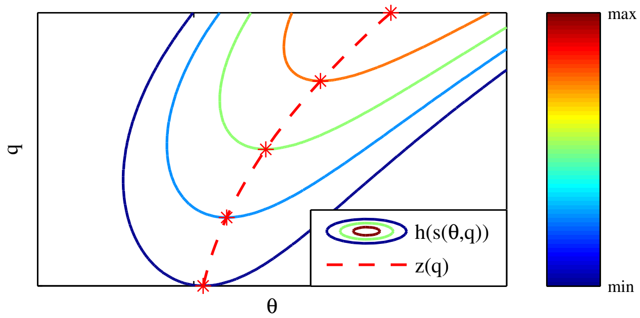

Fig. 1. Illustrations of $h(s(\theta, q))$ and $z(q)$ functions

3. PROPOSED SCHEME

The proposed ESC scheme consists of a performance function feedback and a disturbance feedforward component, see Fig. 2. The feedforward component implements an approximation of the unknown static relation $z(q)$. It computes the feedforward input $\theta_{ff} \in \mathbb{R}$ as a function of the disturbance q and the adaptive feedforward parameters $\eta \in \mathbb{R}^r$. Ideally, one would find the latter by minimizing the norm of the difference between the optimal and the feedforward input. However, as the optimal input θ^* is inherently unknown η is continuously updated using the "best guess" instead, *i.e.*, the (unperturbed) input $\bar{\theta} \approx \theta^*$ to the closed-loop system:

$$\bar{\theta} = \bar{\theta}_{fb} + \theta_{ff}, \qquad (3)$$

where $\bar{\theta}_{fb} \in \mathbb{R}$ represents the unperturbed feedback input produced by the feedback component as a result of the application of the classical gradient-based ESC algorithm of Krstić and Wang [2000].

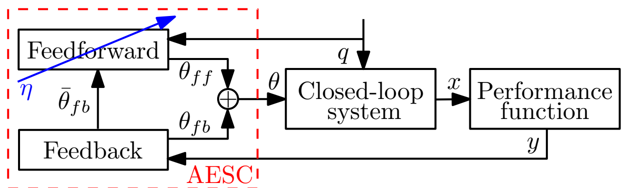

Fig. 2. Adaptive disturbance feedforward ESC

In other words, the feedforward parameters are found by minimizing the square of the approximation error $e \in \mathbb{R}$ given by:

$$e = \bar{\theta} - \theta_{ff} = \bar{\theta}_{fb}. \qquad (4)$$

The input θ is a sum of the (perturbed) feedback and the feedforward input, θ_{fb} and θ_{ff}:

$$\theta = \theta_{fb} + \theta_{ff}, \qquad (5)$$

where the feedback input is obtained by adding a sinusoidal perturbation signal $\delta = \alpha \sin(\omega t)$ to $\bar{\theta}_{fb}$:

$$\theta_{fb} = \bar{\theta}_{fb} + \delta. \qquad (6)$$