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Multi-Model Adaptive Regulation for a Family of Systems Containing Different Zero Structures

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Abstract: An adaptive regulator is proposed for parameter dependent families of linear systems subject to changes in the zero structure. Adaptation is required for the parameter dependent family of plants but continuous adaptive regulation is limited by relative degree and right half plane zeros. A form of adaptive regulation is presented that accommodates parameter induced changes in the zero structure. The conditions for regulation divide the parameter space into disjoint sets thereby defining subfamilies of plants. These plant subfamilies guide controller design. Controller stability is guaranteed by Linear Matrix Inequalities (LMI) and a switch logic based on Lyapunov functions.

Keywords: Regulation; System Structure; Adaptive Control; LMI; Switched Linear system

1. INTRODUCTION

A single controller may be inadequate for systems that experience change in their zero structure. Such systems may be modeled by a structurally diverse family of plants. At any given time the appropriate plant model is uncertain. Multiple model adaptive techniques have been proposed to accommodate such systems Anderson (2000); Angeli and Mosca (2002); Boskovic (2008). Multiple model adaptation selects a controller from a predefined set. In general, the set of controllers is finite although the family of plants may be continuous.

The importance of the open loop zero structure for closed loop regulation has long been known, c.f. Kwakernaak and Sivan (1991); Francis (1977); Kwatny et al. (1991). For a parameter-dependent family of plants, points in the parameter space that do not satisfy the open loop existence conditions for regulator design are called singular points. Singular points form codimension-1 submanifolds that divide the parameter space into disjoint sets. These disjoint sets of the parameter space form subfamilies of plants that have the same zero-structure. A regulator designed for one subfamily will generically fail to regulate a plant in a different subfamily Berg and Kwatny (1994). This bound on *simultaneous regulation* of subfamilies is the basis for the novel multiple model adaptive control design technique presented here.

The design of a finite set of controllers to guarantee stability across the family of plants, called the *covering problem*, is fundamental. Several authors have considered covering from the perspective of controller robustness, Anderson (2000); Boskovic (2008). These designs start with a finite set of plant models and employ robustness metrics to cover the family of plants. We propose a covering method that starts with plant subfamilies and obtains controllers for convex regions of the subfamily's parameter space.

The design of a switch logic to select a stabilizing controller from the set of controllers is the second fundamental problem of multi-model adaptive control. The design method proposed here unifies switch logic and control covering into a single computation. Recall that a single algebraic Riccati equation (ARE) obtains a quadratic Lyapunov function matrix and linear quadratic regulator (LQR) gains. And the quadratic ARE can be written as a convex linear matrix inequality (LMI) to facilitate fast solution. A set of algebraic Riccati inequalities for a convex region of the subfamily's parameter space may be solved for a common LQR state feedback gain and a common quadratic Lyapunov function (CQLF). By choosing Lyapunov function based switch logic and LQR control gains, the multi-model covering and switch logic design computations are unified into a set of LMIs.

This paper is organized as follows. Section 2 defines the specific problem considered herein. Section 3 summarizes the regulation problem and details the relationship between zero dynamics and simultaneous regulation. Section 4 presents our conception of multiple model adaptive regulation (MMAR). Section 5, our main results, details MMAR. Section 6 gives simulation results and Section 7 summarizes the main conclusions.

2. PROBLEM DEFINITION

Define a parameter dependent family of linear plants

$$\dot{x} = A_\theta x + B_\theta u \qquad (1)$$
$$e = C_\theta x$$

as $P(\theta) \in \mathcal{P}$ where $x \in R^n$, $u \in R^m$, $e \in R^p$. The parameter dependent matrices are $A_\theta = A(\theta), B_\theta = B(\theta), C_\theta = C(\theta)$ where $\theta \in R^k$ is a vector of unknown but bounded constant parameters. The goal is to regulate the plant with respect to a set of exogenous signals generated by the model

$$\dot{\vartheta} = Z\vartheta \tag{2}$$

where $\vartheta \in R^r$. The set of exogenous signals considered in this paper are step commands and constant disturbances such that $Z = 0_r$. The exogenous signals are assumed to drive the plant (1) through matrices E and F; accordingly

$$\begin{aligned} \dot{x} &= A_\theta x + B_\theta u + E\vartheta \\ \dot{\vartheta} &= Z\vartheta \\ e &= C_\theta x + F\vartheta \end{aligned} \tag{3}$$

As is well known, such disturbance models can effectively characterize command signals and disturbances. The problem of designing robust regulators for systems described by (3) is well studied, e.g., Davison (1972); Francis (1977); Kwatny and Kalnitsky (1978). In this work an *adaptive* regulator is sought that associates an appropriate robust regulator with the actual occurring member of the plant family. The two central problems in doing this are:

- *Covering Problem*: Given a range of plant parameters θ, design a set of controllers $\mathcal{C}$ such that each $P(\theta) \in \mathcal{P}$ is stabilized by at least one $C_i \in \mathcal{C}$.
- *Switch Logic Design*: Given a plant family $\mathcal{P}$ and a finite control covering, design a switching logic that guarantees convergence to a stabilizing regulator for the actual occurring plant.

In subsequent sections, these issues will be addressed.

3. REGULATION

Before proceeding with adaptive regulation of the parameter dependent system defined in (3) it is necessary to summarize some general results for the regulation of an individual linear system.

3.1 The Linear Regulator Problem

Consider a parameter independent linear system with disturbance state vector ϑ

$$\begin{aligned} \dot{x} &= Ax + Bu + E\vartheta \\ \dot{\vartheta} &= Z\vartheta \\ e &= Cx + F\vartheta \end{aligned} \tag{4}$$

It will be assumed that B and C are of full rank.

Definition 1. Regulation requires both $\lim_{t\to\infty} e(t) = 0$ and internal stability. Regulation in the presence of variation in the plant matrices A, B, C is known as *robust regulation* or *structurally stable regulation.*

Structurally stable regulation uses error feedback and incorporates an internal model of the external signals to be tracked and disturbances to be rejected.

Theorem 2. Francis (1977) Necessary and sufficient conditions for structurally stable regulation are

(1) (A, B) stabilizable
(2) (C, A) detectable
(3) Rank $\begin{bmatrix} \lambda_i - A & B \\ C & 0 \end{bmatrix} = n + r$ for λ_i an eigenvalue of Z

The third condition requires the plant transmission zeros to be different than the spectrum of Z. Furthermore, there must be at least as many controls as there are outputs. Since it is always possible to reduce the number of controls, we will henceforth assume $r = m$, so the system is square.

3.2 Loss of Simultaneous Regulation

Theorem 2 specifies the open loop system $\{A, B, C\}$ for which robust regulation is possible. Now consider robust regulation failure. The system matrix for $\{A_\theta, B_\theta, C_\theta\}$ is

$$\Gamma_\theta(s) = \begin{bmatrix} sI - A_\theta & B_\theta \\ C_\theta & 0 \end{bmatrix}$$

Definition 3. The set of points in parameter space on which regulation fails is the *singular surface*,

$$\left\{\theta \in R^k : \det \Gamma_\theta(0) = 0\right\}$$

The system matrix $\Gamma_\theta(s)$ can lose rank due to a zero at the origin and also due to a defect in the input B_θ or output C_θ matrices. The singular surface is dimension $k-1$, or codimension one in the parameter space. Since Γ_θ is either a regular or singular pencil for fixed θ, the singular surface partitions the parameter space into disjoint sets. Theorem 4 parallels Berg and Kwatny (1994).

Theorem 4. Consider a region of the parameter space bisected by the singular surface. A robust regulator designed for one half of the space will be unstable in the adjacent half space for generic systems.

The singular surface divides the original family of plants into sub-families. A robust regulator designed for (4) and applied to (3) will fail to stabilize adjacent sub-families.

Proof: Loss of simultaneous regulation at a singular surface is introduced in Kwatny et al. (1991) and proved in Berg and Kwatny (1994). Loss of stability at a singular surface for a state feedback regulator design is detailed in Section 5.1.

Traversing a singular surface is a sufficient but not a necessary condition for loss of stability. Loss of stability is certain at the singular surface. Loss of stability is possible within an open region of the parameter space. In summary, the singular surface partitions the parameter space. The resulting disjoint regions are a starting point for multiple model controller selection.

4. MULTIPLE MODEL ADAPTIVE REGULATION

4.1 Covering

Due to Theorem 4, a multiple model approach is employed to regulate the family of plants $\mathcal{P}$. A generic multiple model control structure is illustrated in Figure 1. This generic structure can support numerous control design methods for C_i, even within the same set $\mathcal{C}$.

Here each C_i regulates some region of $\mathcal{P}$ and each $P(\theta) \in \mathcal{P}$ is regulated by at least one $C_i \in \mathcal{C}$. Previous authors, for example Anderson (2000); Boskovic (2008), design controllers for a finite set of plant models and then employ robustness metrics to ensure $\mathcal{P}$ is covered. In this paper, controllers are designed for a finite set of convex polytopes