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Maximum likelihood estimation of aperture jitter using sinusoidal excitation

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ABSTRACT

This paper proposes a solution to estimate the aperture jitter of analog-to-digital (ADCs) converters using the maximum likelihood (ML) approach. The idea is to extend the model built for the ML estimation of ADC and excitation signal parameters to handle the uncertainty of sampling. Using the Gaussian model of aperture jitter this extension requires only one additional parameter. This research article investigates the feasibility of this extension, gives the extended likelihood function in closed form and proposes a slight approximation to decrease computational demand significantly. The increased explanatory power of the extended model is shown via evaluation of simulated and real measurements.

1. Introduction

There are several methods to quantify the imperfection of real analog-to-digital converters. These techniques are estimating some parameters of the device under test and then calculate the static or dynamic quality measures which describe the converter from a certain point of view. Many of these methods are canonized and appear in standards e.g. in [1] or [2]. There are other techniques which can be advantageous in multiple use cases, but are not standardized yet. The maximum likelihood estimation of ADC and excitation signal parameters is one of these. This approach has several attractive properties such as consistency, effectiveness and asymptotic normality [3,4]. On the other hand – owing to its probabilistic approach and the more detailed model – the computation demand is usually larger and the requirements regarding the design of the measurement are more strict in the case of ML estimation compared to the standardized techniques. In conclusion ML estimation is recommended in those cases where the accuracy of estimation is crucial and the number of measurements is large enough to utilize the good asymptotic properties of the ML method. [5] provides a comparative performance analysis of ML and other methods to emphasize the use cases where ML estimation is recommended to achieve satisfying ADC test results.

The model developed for ML estimation of ADC and excitation signal parameters [6] assumes ideal sampling, thus does not handle aperture jitter. This approach can be useful in several cases where the stimulus is slow and noisy thus the effect of aperture jitter can be neglected compared to the effect of the additive noise. However, in the case of a faster and less noisy signal the aperture jitter becomes significant and results a time-variant, amplitude-modulated noise component. Modeling the uncertainty of sampling has two important advantages: on the one hand it increases the explanatory power of the model, on the other hand it provides an ML estimate for the aperture jitter which can be important for the users of the ADC under test. The aperture jitter can be estimated different ways besides the ML method as it is described in [7,8] or [9]. Standard IEEE-1241 [1] also proposes a method to measure aperture jitter, but it requires a special measurement setup dedicated to estimate the amount of jitter. Nevertheless these estimation methods do not provide the attractive properties that the ML estimation does. Furthermore the suggested ML method estimates the jitter using a simple sinusoidal measurement record while also estimates multiple other parameters of the stimulus and the device under test – thus makes possible to calculate dynamic quality measures such as ENOB or SINAD. This paper proposes an extension of the model developed for ML estimation which handles – and thus estimates in ML sense – the aperture jitter while the parameter space grows only by one additional parameter. The details of this extension are itemized in the following sections.

2. The model

The measurement setup for sinusoidal ADC testing is simple. A dedicated sine wave generator is connected to the input of the ADC under test that records this stimulus. It is recommended to use a slow ($f \approx 10$ Hz...100 Hz) sine wave, the length of the record is usually a few seconds. Depending on the sampling rate this measurement leads to a digital record containing a few hundred thousand or a few million samples. Since noise is part of the model – the amount of noise is also a parameter to be estimated – EMC protection and avoiding disturbances is not particularly important. Naturally more noise leads to larger variance in the estimation, thus it is recommended to pursue low-noise

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environment, but not crucial. The measurement noise is consequence of different phenomena. The main components of it are.

- the electronic noise of the sine wave generator,
- the electromagnetic disturbances superimposed to the signal due to the cable and
- the elctronic noise of the amplifier in the sample and hold subcircuit of the ADC.

Note that these three categories cover many individual noise sources: the electronic noise is result of several different semiconductor components and the EM disturbances are due to multiple devices (transformers, switch mode power supplies, etc.). Owing to the different sources of disturbance the noise profile of the measurements is largely balanced: the presence or absence of a certain noise source does not have large influence on the overall noise profile.

The record of the sampled and quantized noisy sine wave is being processed during the evaluation. The parameters of the model — according to the notation used in [6] – are the following:

- **Parameters of the sinusoidal excitation:** $x(t) = A\cos(2\pi ft) + B\sin(2\pi ft) + C$ and $x_n(t) = x(t) + n(t)$, where A is the cosine coefficient, B is the sine coefficient of the sine wave, C stands for the DC component of the stimulus and the frequency is denoted by f. The additive noise $n(t)$ is assumed to be Gaussian white noise [6] with zero mean and σ standard deviation. Note that the frequency is usually expressed as sampled angular frequency: $\theta = 2\pi fT_s$, where T_s is the nominal sampling time.
- **Parameters of the quantizer:** the nonideal quantizer is described by its code transition levels. The k^th^ code transition level is the DC voltage which results output code $k-1$ with 50 % probability and code k with 50 % probability when applied to the input of the quantizer. An N-bit quantizer has 2^N-1 code transition levels, these are denoted by $T_1, T_2, \ldots, T_{2^N-1}$.

Therefore the parameter vector is

$$\mathbf{p}^T = [A \quad B \quad C \quad \theta \quad \sigma \quad T_1 \quad \ldots \quad T_{2^N-1}]. \tag{1}$$

To express the likelihood of a given parameter set we have to introduce a discrete random variable $Y[k]$. This random describes gives the probability distribution of the k$^{\text{th}}$ sample assuming the parameters contained by $\mathbf{p}$. The $\mathrm{P}[Y[k] = l]$ quantity is the probability of the incidence the k$^{\text{th}}$ sample noisy sine wave $x_n(t)$ is between T_l and T_{l+1}. Owing to the Gaussian noise model we have to use the error function to express these probabilities:

$$\mathrm{erf}(x) = \int_0^x e^{-t^2}\mathrm{d}t. \tag{2}$$

Hence the probability distribution of the discrete random variable $Y[k]$ can be expressed as

$$\mathrm{P}[Y[k] = l] = \frac{1}{2}\left[\mathrm{erf}\left(\frac{T_{l+1}-x[k]}{\sigma\sqrt{(2)}}\right)-\mathrm{erf}\left(\frac{T_l-x[k]}{\sigma\sqrt{(2)}}\right)\right], \tag{3}$$

where $x[k]$ is the k$^{\text{th}}$ sample of the noiseless sine wave: $x[k] = A\cos(k\theta) + B\sin(k\theta) + C$. Note that this model assumes ideal, equidistant sampling. Exploiting the independence of the noise samples the likelihood function of the measurement is

$$\mathrm{L}(\mathbf{p}) = \prod_{k=1}^{M} \mathrm{P}[Y[k] = y[k]], \tag{4}$$

where $y[k]$ is the k$^{\text{th}}$ sample of the digital measurement record. Using (3) and (4) the likelihood function can be written as

$$\mathrm{L}(\mathbf{p}) = \prod_{k=1}^{M} \frac{1}{2}\left[\mathrm{erf}\left(\frac{T_{y[k]+1}-x[k]}{\sigma\sqrt{(2)}}\right)-\mathrm{erf}\left(\frac{T_{y[k]}-x[k]}{\sigma\sqrt{(2)}}\right)\right]. \tag{5}$$

The objective function to be optimized is usually the negative log-likelihood function which is also known as cost function:

$$\mathrm{CF}(\mathbf{p}) = -\ln\mathrm{L}(\mathbf{p}) = M\ln 2 - \sum_{k=1}^{M} \ln\left[\mathrm{erf}\left(\frac{T_{y[k]+1}-x[k]}{\sigma\sqrt{(2)}}\right) -\mathrm{erf}\left(\frac{T_{y[k]}-x[k]}{\sigma\sqrt{(2)}}\right)\right]. \tag{6}$$

Using the log-likelihood function is advantageous regarding the optimization process since the calculation of the derivatives becomes less complex and numerically less tense.

3. Extension of the ML estimation framework to model nonideal sampling

The phenomena owing to nonideal sampling are not considered in the model originally for ML estimation of sine wave and ADC parameters. The original article [6] introduces the additive noise on the analog excitation as a parameter to estimate and the effect of nonideal sampling – which can be handled using the term aperture jitter – is not separated from the additive noise on the analog signal. The uncertainty of sample $y[k]$ owing to the aperture jitter is

$$\Delta y[k] \simeq \left.\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right|_{t=t_{k,\mathrm{ideal}}} \cdot \Delta t_k, \tag{7}$$

where $x(t)$ is the noiseless sinusoidal excitation signal, $t_{k,\mathrm{ideal}}$ is the ideal sampling time corresponding to the k$^{\text{th}}$ sample, t_k is the real sampling moment of t_k and Δt_k is the difference of the real and the ideal moment of sampling for the k$^{\text{th}}$ sample: $\Delta t_k = t_k - t_{k,\mathrm{ideal}}$. Assuming that the Δt_k values can be characterized by the same probability distribution the additive noise owing to aperture jitter can be modeled as amplitude-modulated noise. Fig. 1 illustrates this model: the Δt_k values are independent realizations of a random variable and $\Delta y[k]$ values are calculated as described in (7).

The standard deviation of the additive noise owing to the jitter can be calculated as

$$\sigma_{n,t} = \left.\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right|_{\mathrm{rms}} \cdot \sigma_t, \tag{8}$$

where σ_t is the standard deviation of the distribution that provides the Δt values. Let us introduce μ_t for the expected value of this distribution. Since the sampling usually does not have systematic error components, we assume $\mu_t = 0$. In (8) $\left.\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right|_{\mathrm{rms}}$ is the root mean square value of the slope of the noiseless excitation signal. Since $\frac{\mathrm{d}x(t)}{\mathrm{d}t}$ is sinusoidal as well, its RMS value is

$$\left.\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right|_{\mathrm{rms}} = \frac{2\pi fA}{\sqrt{2}}, \tag{9}$$

where A is the amplitude and f is the frequency of $x(t)$. Since the excitation signals used for ADC testing are usually slow sine waves ($f < 100\,\mathrm{Hz}$) and the aperture jitter of the modern S/H circuits is low (typically a few picoseconds and very rarely larger than 50 ps, see [10]) the additive noise owing to jitter can be majorized as

$$\sigma_{n,t,\mathrm{max}} = \left.\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right|_{\mathrm{rms,max}} \cdot \sigma_{t,\mathrm{max}} = \frac{2\pi\cdot 100\mathrm{Hz}\cdot 10\mathrm{V}}{\sqrt{2}}\cdot 50\mathrm{ps} = 0.2221\mu\mathrm{V}. \tag{10}$$

This amount of noise is usually negligible compared to the analog noise owing to the semiconductor devices and the EM interference. This way on the one hand it is valid to assume that the effect of jitter can be neglected – it is a component of the additive noise thus it can be integrated into the additive noise model. On the other hand these ADCs shall also measure signals which are faster by several orders of magnitude and jitter is not only a random error component but can cause systematic errors (as it is shown in Section 3.1). Therefore it worths to