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A note on improvement of adaptive observer robustness

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Abstract: In this paper the problem of adaptive observer design in the presence of disturbances is studied, and an augmented adaptive observer is proposed. First, the H_∞ gain of a conventional adaptive observer is estimated, which characterizes the effect of disturbances on output errors. Next, it is shown that if the disturbance is "matched" in the plant equations, then it is possible to introduce additional sliding-mode feedback, dependent on the plant output, improving the accuracy of observation. Simulation results confirm the improvement.

1. INTRODUCTION

The problem of adaptive observer design for nonlinear systems is very challenging Besanon [2007]. In this situation the observer has to generate estimates of the vector of unknown parameters and unmeasured state components. Almost all solutions are based on special canonical representations of the uncertain nonlinear systems, that allows the observer to be designed. An important obstacle is the *relative degree* between the output signal and the vector of unknown parameters (i.e. the number of derivatives of the output required, before the direct dependence on the vector of unknown parameters is obtained). The observers designed in the case when the degree is one Fradkov et al. [2000] and for the higher relative degree case Fradkov et al. [2002], Xu and Zhang [2004], Zhang [2002], Efimov [2004] have completely different structures, and the dimension of the observers in the latter case is much higher.

The structure of the observer proposed in Zhang [2002] for the high relative degree case is quite sophisticated, and strongly related to a canonical form of the plant equations. A small modification to the plant structure may render the observer design impossible. To overcome this limitation, a deviation of equations of the real system from the canonical form can be modeled by external disturbances. These disturbances have to be bounded, but in some situations it is hard to assume that the disturbance is sufficiently small. Thus it is necessary to ensure robustness of the designed observers with respect to such a disturbance.

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There exist a number of solutions aimed at improving the robustness in nonlinear systems by applying dynamic or static feedback. Some very promising solutions have been obtained in the area of sliding mode theory since sliding mode feedback is able to fully compensate for matched disturbances even granting the closed loop system finite-time stability Shtessel et al. [2013]. Recently the sliding mode approach has been successfully applied to adaptive observer design for the case of relative degree one Yan and Edwards [2008], but the application of this theory for adaptive observer design with a high relative degree is somewhat complicated due to the fixed observer structure. In this paper a method is presented for modifying the adaptive observer from Zhang [2002] using a sliding-mode feedback in the spirit of Yan and Edwards [2008], that allows us to improve the level of observer robustness with respect to some matched disturbances.

The paper is structured as follows. The adaptive observer equations and the problem statement are given in Section 2. The H_∞ gain of the adaptive observer from Zhang [2002] is computed in Section 3. Then the sliding-mode feedback methodology is applied in Section 4. An example of a computer simulation (for a nonlinear pendulum) is presented in Section 5 to demonstrate the efficacy of the approach.

2. PROBLEM STATEMENT

Consider the following uncertain nonlinear system:

$$\dot{x} = Ax + \phi(y, u) + G(y, u)\theta + Bv, \; y = Cx, \qquad (1)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$, $u \in \mathbb{R}^m$ are the state, the output and the control respectively, $\theta \in \mathbb{R}^q$ is the vector of unknown parameters, $v \in \mathbb{R}^s$ is the vector of external disturbances and $v : \mathbb{R}_+ \to \mathbb{R}^s$ is a (Lebesgue) measurable function of time; the matrices A, B, C are known and have corresponding dimensions, the functions $\phi : \mathbb{R}^{p+m} \to \mathbb{R}^n$ and $G : \mathbb{R}^{p+m} \to \mathbb{R}^{n\times q}$ are also assumed to be known and ensure uniqueness and existence of the system (1) solutions at least locally.

The symbol $|x|$ denotes the Euclidean norm of a vector x (for a matrix A the symbol $|A|$ denotes the induced matrix norm), and for the (Lebesgue) measurable functions $v : \mathbb{R}_+ \to \mathbb{R}^s$, the norm is defined as $||v|| = ess\, sup_{t\geq 0}\{|v(t)|\}$. For a matrix function $A : \mathbb{R}_+ \to \mathbb{R}^{s\times q}$ we denote $||A|| = |||A(t)|||$. The identity matrix of dimension $n \times n$ is denoted as I_n. The symbols $\lambda_{\min}(A)$, $\lambda_{\max}(A)$ represent the minimal and maximal eigenvalues of a symmetric matrix $A \in \mathbb{R}^{n\times n}$.

In this work we will assume that all external and internal signals in the system (1) are bounded:

Assumption 1. $||x|| < +\infty$, $||v|| < +\infty$, $||u|| < +\infty$.

Although assumed to be bounded, the disturbance v may have a large amplitude, and therefore special attenuation techniques have to be applied to ensure reliable estimates for the states in system (1).

The objective of this work is to design an adaptive observer for (1) under the conditions of Assumption 1. The observer has to provide estimates of the vectors x and θ with an improved degree of robustness with respect to the external disturbance v. The proposed design procedure is completed in two steps. Firstly, an adaptive observer is designed such that the H_∞ gain between the disturbances and the output errors is calculated (and thus it can be minimized). Secondly, an additional sliding mode output injection is applied to further reduce the influence of the disturbance components which cannot be completely rejected by the first step. The proof of stability is given for each case. An analysis of the structure of the resulting observers suggests that the augmented adaptive observer behaves better than the conventional adaptive observer with H_∞ gain optimization. The simulation results confirm this.

An adaptive observer for the system (1) has been proposed in Zhang [2002], and takes the form:

$$\dot{z} = Az + \phi(y,u) + G(y,u)\hat{\theta} + L(y - Cz) + \Omega\dot{\hat{\theta}}, \qquad (2)$$

where

$$\begin{aligned}\dot{\Omega} &= (A - LC)\Omega + G(y,u),\\ \dot{\hat{\theta}} &= \gamma\Omega^T C^T(y - Cz).\end{aligned} \qquad (3)$$

In (2)–(3), $z \in \mathbb{R}^n$ is the estimate of x, $\hat{\theta} \in \mathbb{R}^q$ is the estimate of θ, and $\Omega \in \mathbb{R}^{n\times q}$ is an auxiliary/filter variable, that helps overcome possible high relative degree obstructions in system (1). In (3) $\gamma > 0$ is a design parameter, and L is the observer gain that is chosen to ensure a Hurwitz property for the matrix $A - LC$. The analysis of the estimation abilities of the observer in (2), (3) is based on the errors $\delta = x - z + \Omega\tilde{\theta}$ and $\tilde{\theta} = \hat{\theta} - \theta$ whose dynamics can be shown to have the form:

$$\dot{\delta} = (A - LC)\delta + Bv, \qquad (4)$$

$$\dot{\tilde{\theta}} = \gamma\Omega^T C^T(C\delta - C\Omega\tilde{\theta}). \qquad (5)$$

From equation (4) we conclude that $\delta \to 0$ for $v = 0$ and the variable δ stays bounded for any bounded disturbance v. The Hurwitz property of the matrix $A - LC$ and Assumption 1 imply boundedness of the variable Ω. If the signal $G(y,u)$ is persistently exciting (PE) Anderson [1977], Yuan and Wonham [1977], then due to the filtering property of the variable Ω, the variable $C\Omega$ is also PE. Moreover, it is possible to show Efimov [2004] that for any bounded signal $C\delta$, the variable $\tilde{\theta}$ has a bounded response, and if $C\delta \to 0$, then $\tilde{\theta} \to 0$ also. Since as a consequence of Assumption 1 the variables x, δ, $\tilde{\theta}$ and Ω are bounded, the variable z is also bounded according to the definition of δ. In addition, it is easy to verify that if $\tilde{\theta} \to 0$, then $z \to x$. Thus, we have shown that the variables z, Ω and $\hat{\theta}$ stay bounded for any bounded disturbance v, and for the case when $v = 0$ the estimates $\hat{\theta}$ and z converge to their ideal values θ and x respectively. Thus the proof is based on general stability arguments, and no strict Lyapunov function has been proposed.

To improve the accuracy of estimation in the observer (2)–(3), one can design the matrix L optimizing the H_∞ gain of the system $(C, A - LC, B)$ Henry and Zolghadri [2005]. In this case the influence of the disturbance v on the output $C\delta$ of the system in (4) is minimized, and implies a proportional amelioration of the error $\tilde{\theta}$ in (5). This will be proven formally in Section 3 below. Moreover, it will be shown that if v is a Lipschitz function of x, then by proper selection of L it is possible to attenuate the disturbance v. If the amplitude of the disturbance v is large, then such an optimization could be insufficient. This problem can be addressed by modifying equation (2): this approach is later demonstrated in Section 4 using a sliding mode technique.

3. CONVENTIONAL ADAPTIVE OBSERVER

First let us show that the system in (5) is input-to-state stable with respect to the input $C\delta$.

Lemma 1. Let the variable $\Omega^T C^T$ be PE and bounded, i.e. $0 < \rho = ||C\Omega|| < +\infty$ and there exist constants $\vartheta > 0$ and $\ell > 0$ such that

$$\int_t^{t+\ell} \Omega(\tau)^T C^T C\Omega(\tau)d\tau \geq \vartheta I_q \quad \forall t \geq 0.$$

Then

a) there exists a continuous symmetric matrix function $P : \mathbb{R}_+ \to \mathbb{R}^{q\times q}$ such that $\rho^{-2}I_q \leq 2\gamma P(t) \leq \alpha I_q$ for all $t \geq 0$, where $\alpha = \gamma\eta^{-1}e^{2\eta\ell}$ and $\eta = -0.5\ell^{-1}\ln(1 - \frac{\gamma\vartheta}{1+\gamma^2\ell^2\rho^4})$;

b) for all $t \geq 0$

$\dot{P}(t) - \gamma P(t)\Omega(t)^T C^T C\Omega(t) - \gamma\Omega(t)^T C^T C\Omega(t)P(t) + I_q = 0;$

c) for $S(t,\tilde{\theta}) = \tilde{\theta}^T P(t)\tilde{\theta}$ we have for all $\tilde{\theta} \in \mathbb{R}^q$, $\delta \in \mathbb{R}^n$ and $t \geq 0$

$$\dot{S} \leq -\gamma\alpha^{-1}S + 0.5\rho^2\alpha^2|C\delta|^2.$$

In addition, for all $\tilde{\theta}(0) \in \mathbb{R}^q$ and all $t \geq 0$ the following estimate is satisfied:

$$|\tilde{\theta}(t)| \leq \rho\sqrt{\alpha}[e^{-0.5\gamma\alpha^{-1}t}|\tilde{\theta}(0)| + \rho\alpha||C\delta||].$$

The proof is omitted for brevity.