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Switched observers for state and parameter estimation with guaranteed cost

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Abstract: This paper addresses the problem of state and parameter estimation for the class of affine systems in the state space representation. The method does not require a specific state representation of the system and consists of designing a switched observer that, under certain conditions given in the paper, allows for the state and parameter estimation errors to converge to zero. Assuming that the parameters to be estimated belong to a given polytope, the idea of the method is to recast the parameter estimation problem as a switching rule design for an auxiliary switched system whose matrices at the equilibrium correspond to the matrices of the system to be estimated. A guaranteed cost is used in the design and the switching rule is based on a max composition of a set of quadratic functions of the observation error. The method is simple and has low computational cost. The main disadvantage regards the amount of information that is needed to have both state and parameters estimated simultaneously. The case when there is no parameter to estimate the method reduces to a standard Luenberger observer with guaranteed cost.

Keywords: State Observers, Guaranteed Cost, Parameter Estimation, Switched Systems, Sliding Mode, Lyapunov Function.

1. INTRODUCTION

In many practical applications, the situation where all state variables are available from measurement is not realistic. A similar situation occur with the parameters of the system, and usually, some of them are not precisely known. Pointed by Postoyan et al. (2012), despite of its importance, the problem of estimating simultaneously the state and parameters of the system is not widely explored as a single problem.

Although some of the current approaches for both state observation and parameter identification can be applied to a wide class of problems (Farza et al., 2009), even in the non-linear case, for example Zhou et al. (2013); Farza et al. (2009). These methods usually lead to complex observer structures found in Zhou et al. (2013); Fridman et al. (2006), or require a large number of auxiliary variables to be estimated Farza et al. (2009). Adaptive observers are often based on a transformation of the original system into some canonical form in which the presence of the unknown parameter is simplified to some extent in Zhang (2005). Few adaptive observers in non canonical form can be found in the literature, for instance Zhang (2005); Tyukin et al. (2009). As described in Fridman et al. (2006), the output injection for both linear feedback or sliding mode is commonly used, but a kind of persistent excitation condition is required and, in general, this condition cannot be easily checked.

In this paper we propose a switched observer approach to cope with the problem of simultaneously estimating the states and parameters of the system. The proposed observer is composed of a set of subsystems, an observer gain and a switching rule. The matrices of the subsystems are obtained from the vertices of the polytope that define the bounds on the parameters to be estimated. The observer gain consists of a switched gain matrix and the switching rule is determined in order to guarantee the convergence of the state and parameter errors to zero with guaranteed cost performance. The sliding mode dynamics of the switched system are represented according to Filippov's results (Filippov, 1988, p. 50). An LMI approach is proposed to solve the problem, i.e. to find the observer gain and the switching rule. The LMI is dependent on the observer state and its feasibility requires the observer state trajectory does not leave a given polytope representing a bound on the observer state. The vertices of this polytope can be adjusted to met this requirement and this condition plays the role of well known persistent excitation requirements found in usual adaptive schemas for parameter estimation problems. The paper is organized as follows. After a notation paragraph, the next section is devoted to some preliminaries and to present the problem formulation. The main result is presented in the Section 3. The results are illustrated in the section 4 through a mechanical system and some concluding remarks end the paper. This paper

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is an extension of Grala Pinto and Trofino (2014) in the sense that a guaranteed cost performance is now included in the design problem.

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices; $\|\cdot\|$ stands for the Euclidian norm of vectors and its induced spectral norm of matrices; $\star$ represents block matrix terms that can be deduced from symmetry; 0_n and $0_{m\times n}$ are the $n \times n$ and $m \times n$ matrices of zeros, I_n is the $n \times n$ identity matrix. $\mathbf{1}_n$ and $\mathbf{1}_{m\times n}$ are matrices of dimension $n \times n$ and $m \times n$ where all entries are the unity. For a real matrix S, S^T denotes its transpose and $S > 0$ ($S < 0$) means that S is symmetric and positive-definite (negative-definite). For a set of real numbers $\{v_1, \dots, v_m\}$ we use arg $\max\{v_1, \dots, v_m\}$ to denote a set of indexes that is the subset of $\{1, \dots, m\}$ associated with the maximum element of $\{v_1, \dots, v_m\}$. $\lambda_{max}(.)$ and $\lambda_{min}(.)$ denotes the maximum and minimum eigenvalue of a symmetric matrix. For a set of integers $\mathcal{M}$ the notation $\mathcal{P}(\mathcal{M})$ denotes its power set.

2. PROBLEM STATEMENT

Consider the system

$$\dot{x}(t) = Ax(t) + b + Dr(t) \quad , \quad y(t) = Cx(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^{n_y}$ is the measurement vector and the input $r(t) \in \mathbb{R}^{n_r}$ is a known excitation signal. It is assumed that the system is exponentially stable. The matrices $C \in \mathbb{R}^{n_y\times n}$ and $D \in \mathbb{R}^{n\times n_r}$ are assumed to be known. The matrix $A \in \mathbb{R}^{n\times n}$ and vector $b \in \mathbb{R}^n$ are unknown and it is assumed that (A, b) is an element (to be found) of the convex hull

$$(A, b) \in \underset{i\in\mathcal{M}}{\mathbf{Co}}\{(A_i, b_i)\} = \sum_{i=1}^{m} \theta_i (A_i, b_i)$$

where $\underset{i\in\mathcal{M}}{\mathbf{Co}}\{.\}$ denotes the convex hull, $\mathcal{M} := \{1, \dots, m\}$ is a set of integers, (A_i, b_i) are given matrices, and the vector $\theta = [\theta_1 \; \dots \; \theta_m]^T$ is an element of the m-dimensional unity simplex

$$\Theta = \left\{ \theta \in \mathbb{R}^m : \theta_i \geq 0, \; \forall i \text{ and } \sum_{i=1}^{m} \theta_i = 1 \right\},$$

From now on we use the notation

$$A = A_{\bar\theta} = \sum_{i=1}^{m} \bar\theta_i A_i, \qquad b = b_{\bar\theta} = \sum_{i=1}^{m} \bar\theta_i b_i, \tag{2}$$

where $\bar\theta \in \Theta$ is a parameter to be found, characterizing the system matrices A, b of (1) as an element of the above convex hull. Observe $A_{\bar\theta}$ is Hurwitz from assumption.

Problem 1. Given the matrices A_i, b_i, C, D, the measurement signal $y(t)$ and the input signal $r(t)$, for $t \geq 0$, the problem of concern is to find a switching rule and observer gains L_i such that the following switched observer

$$\dot{z}(t) = A_\theta z(t) + b_\theta + L_\theta(y(t) - Cz(t)) + Dr(t) \tag{3}$$

$$(A_\theta, b_\theta, L_\theta) = \sum_{i=1}^{m} \theta_i(x, z)(A_i, b_i, L_i) \tag{4}$$

satisfies the following convergence properties:

$$\lim_{t\to\infty} z(t) = x(t) \quad , \quad \lim_{t\to\infty} \theta(x(t), z(t)) = \bar\theta \tag{5}$$

and minimizes an upper bound of the following guaranteed cost

$$J = \min_{L_\theta, \sigma(\varepsilon)} \; \max_{\substack{e_0\in\mathcal{E}_0,\\ \bar\theta,\theta\in\Theta}} \int_0^\infty \xi^T(t)\, \xi(t)\, dt \tag{6}$$

where $\theta(x, z) \in \Theta$ is a piecewise continuous multivalued function defined according to Filippov's results (Filippov, 1988, p. 50) for discontinuous right hand side equations. $\mathcal{E}_0$ denotes a given set of initial conditions $e_0 = x(0) - z(0)$ and $\xi(t)$ is the performance output

$$\xi(t) = C_p e(t) + D_p \delta(t) \tag{7}$$

where C_p, D_p are given weighting matrices, and

$\varepsilon(t) = y(t) - Cz(t)$, $e(t) = x(t) - z(t)$, $\delta(t) = \bar\theta - \theta(z(t), x(t))$

are, respectively, the output estimation error, state estimation error and parameter estimation error, and the switching rule is represented by a piecewise constant set valued function $\sigma(\varepsilon(t)) \subseteq \mathcal{P}(\mathcal{M})$. □

Note that $\varepsilon = Ce$. Recall that when $\sigma(\varepsilon(t))$ is a singleton, namely when $\sigma(\varepsilon(t)) = \{i\}$, the parameter $\theta(x(t), z(t))$ is such that $\theta_i(x(t), z(t)) = 1$ and thus $\theta_j(x(t), z(t)) = 0, \; \forall j \neq i$. When $\sigma(\varepsilon(t))$ is not a singleton and a sliding mode is occurring, the role of $\theta(x(t), z(t))$ is to keep the system vector field on the tangent hyperplane of the switching surface where the sliding motion is taking place. See (Filippov, 1988, p. 50) for details.

Note that when $\sigma(\varepsilon(t)) = \{i\}$ is a singleton the observer (3) takes the form

$$\dot{z}(t) = A_i z(t) + b_i + L_i (y(t) - Cz(t)) + Dr(t) \tag{8}$$

From the states of the system, the observer and the decompositions (2), (4) we get the dynamics of the estimation error as follows.

$$\dot{e}(t) = (A_{\bar\theta} - L_\theta C)\, e(t) + (A_{\bar\theta} - A_\theta)\, z(t) + (b_{\bar\theta} - b_\theta) \tag{9}$$

Recall that $\bar\theta$ is an unknown element of the unity simplex.

In order to estimate $\bar\theta$ and the system state with the convergence properties (5), let us consider the following set of m auxiliary functions $v_i(e, \bar\theta)$.

$$\begin{aligned} v_i(e, \bar\theta) &= e^T C^T P_i C e + 2e^T C^T S_i - 2e^T C^T S_{\bar\theta} + e^T Q_{\bar\theta} e \\ &= \varepsilon^T P_i \varepsilon + 2\varepsilon^T S_i - 2\varepsilon^T S_{\bar\theta} + e^T Q_{\bar\theta} e \end{aligned} \tag{10}$$

where $S_{\bar\theta} = \sum_{i=1}^{m} \bar\theta_i S_i$, $Q_{\bar\theta} = \sum_{i=1}^{m} \bar\theta_i Q_i$.

Based on the above auxiliary functions $v_i(e, \bar\theta)$ consider a switching rule characterized by the set-valued function

$$\begin{aligned} \sigma(\varepsilon) &= \arg\max_{i\in\mathcal{M}} \{v_i(e, \bar\theta)\} \\ &= \arg\max_{i\in\mathcal{M}} \{\varepsilon^T P_i \varepsilon + 2\varepsilon^T S_i\} \end{aligned} \tag{11}$$

Observe that despite the dependence of v_i with respect to $(e, \bar\theta)$, the switching signal σ is a function of ε only.

The following result, known as Finsler's Lemma, found in Boyd et al. (1997), is of interest to this paper.

Lemma 1. (Finsler Lemma). Let $\mathcal{W} \subseteq \mathbb{R}^s$ be a given polytopic set, $M(.) : \mathcal{W} \longmapsto \mathbb{R}^{q\times q}$, $G(.) : \mathcal{W} \longmapsto \mathbb{R}^{r\times q}$ be given matrix functions, with $M(.)$ symmetric. Let $Q(w)$ be a basis for the null space of $G(w)$. Then the following are equivalent: