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Transient responses and stability in the differential electrostatic sensor of inertial and gravitational moments with asymmetry

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ABSTRACT

High sensitivity of Gravity Inertial Sensors (GIS) can be achieved with low natural frequency Ω of the proof mass (PM) that must have low level noises in the read-out system and a minimized torsion stiffness. In this article, a differential electrostatic system (DES) is proposed where both previous two conditions are met. We also consider the effect of the asymmetry γ of the DES in the GIS that is limited by the manufacturing technology of the sensor and hence making it difficult to reduce further. We propose an alternative compensation of this asymmetry by introducing additional sources of electric field. It is shown that any inaccuracy in this compensation will lead to inaccuracy of the sensor and can violates it stability. Based on open public literature, this is the first time where it is possible to ascertain the torsion stiffness reduction limits for such a sensor.

1. Introduction

Gravity Inertial Sensors (GIS), e.g. linear and angular accelerometers, seismographs and gravity gradiometers having the highest possible sensitivity, are usually based on an elastically suspended proof mass (PM) rotating with respect to the frame within a small angle around a fixed axis while they are measuring [1–6]. Such sensors must have low natural frequency and low noise levels. However, they are all limited by the asymmetry due to manufacturing and no alternative methods to compensate for it have been suggested. Thermal and non-thermal noises occur within the sensor in the external pick-off circuits and in the feedback circuits. These observations affect the dynamic properties of the sensors [7,8]. On the other side, capacitive sensors can provide the required noise level and sensitivity [9]. Recently, much attention was given to resonant sensors in which an electrostatic field is used to reduce the torsional rigidity of the torsion suspensions [10,11]. A simple capacitive GIS was proposed in [12]. This sensor does not have resonance circuit to read useful signals and feedback circuits. The differential electrostatic system (DES) is used in this GIS to obtain a low natural frequency. A distinctive feature of the sensor is that the electrostatic read-out system is used to reduce the natural frequency. It is assumed that such a sensor has the lowest possible noise level. It was considered previously that the natural frequency of the PM in the DES can be arbitrarily small [13]. This can be affected by manufacturing issues such as asymmetry of the PM, or non-parallelism of the capacitance plates as discussed in [20]. Some attempts to solve these problems were triggered without much success e.g. the known electrostatic effect «pull-in» that arises when attempting to increase the electrostatic field, so that the system becomes unstable and the PM sticks to one of the electrodes [16]. Application of GIS in innovative small and large products with embedded control is extremely important for foresight innovation [18].

Based on this review, we consider in this paper to compensate for the effect of the asymmetry of the DES in the GIS caused by the manufacturing technology limitation of the sensor. We have already shown in [14,15] that the asymmetry of the DES is the factor limiting the ability of reducing the natural frequency without further treatment. The proposed approach is to analyze both linear and nonlinear dynamics of GIS with DES having an asymmetry in conditions of free and forced PM oscillations, and to analyze possibilities to compensate the effects of the asymmetry by introducing additional sources of electric field. The reduction of torsion stiffness becomes possible leading to the enhancement of the DES sensitivity.

The paper demonstrates, for the first time, the influence of permittivity on the sensor's accuracy, the influence of the permittivity and the frequency ratio on sensor's stability and forced oscillations. The paper derives explicit relationships determining the minimum of torsional stiffness that may be reached in real sensors.

2. Description of the sensor's electrical circuit

Let us consider the sensor circuit (Fig. 1) described by a differential

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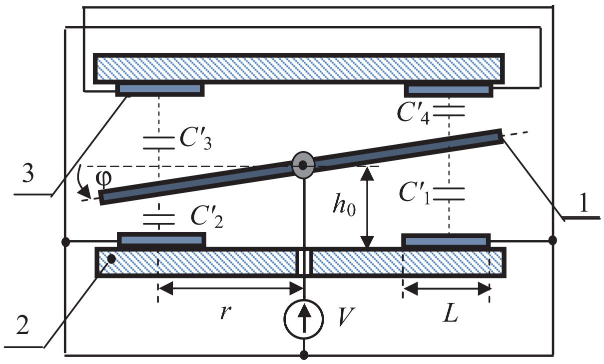

Fig. 1. The “three-layer” differential sensor model. 1- conductive PM, 2- non-conductive plate, 3- electrodes. PM is shown with an elastic torsion in it center.

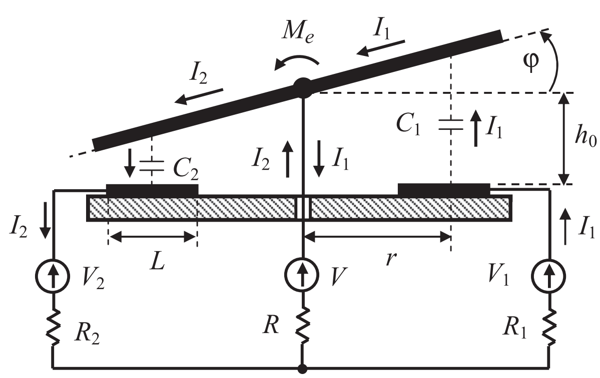

Fig. 2. The equivalent “double-layer” model of the DES that takes into account the presence of additional resistors and constant voltage sources.

capacitive signal read-out system, called “three-layer capacitive sensor” [1]. It is assumed that the capacitance components are related as $C'_1 = C'_3 = C_0$ and $C'_2 = C'_4 = (1 + \gamma)C_0$, where γ is the asymmetry parameter of the DES. Taking into account that capacitances C'_1 and C'_3 and capacitances C'_2 and C'_4, are connected in parallel and introducing notations; $\gamma_1 = 1 + \gamma$ and $2C_0 = C_s$, the three layer circuit can be reduced to an equivalent “double-layer” circuit as shown in Fig. 2 [2]. When analyzing the “double-layer” scheme, it will take into account the presence of additional resistors and DC voltage sources.

In the circuit of Fig. 2, the dependences of the capacitances with the rotation angle φ of the PM rotation are given by [15] in Eqs. (1) & (2).

$$C_1(\varphi) = \frac{C_s\varphi_m}{\varphi_m + \varphi}, C_2(\varphi) = \frac{\gamma_1 C_s \varphi_m}{\varphi_m - \varphi}, \text{where } C_s = \frac{2\varepsilon_0 S}{h_0}, \tag{1}$$

$$\phi_m = \frac{h_0}{L}\ln\frac{a_{L2}}{a_{L1}}; a_{L1} = r - \frac{L}{2}; a_{L2} = r + \frac{L}{2}. \tag{2}$$

3. Non-linear and linear mathematical models of the sensor

It is well known that the moment $M(\varphi)$ of the forces of electric attraction between the electrodes, as a function of rotation angle, can be determined in terms of the energy of electric field $W_e(\varphi)$ in the capacitor. Hence, in this case only, the capacitors depend explicitly on the angle φ. The relation between the torque and the energy is expressed by Eq. (3).

$$M(\varphi) = -\frac{\partial W_e(\varphi)}{\partial \varphi} \tag{3}$$

where $W_e(\varphi) = -\frac{C(\varphi)V^2}{2}$ and V is the applied voltage.

Using Eqs. (1–3), the moments $M(\varphi)$ in the capacitors C_1 and C_2 can be found. Taking into account future calculations, it is convenient to express the moments as function of charges q_1 and q_2 of the corresponding capacitors:

$$M_1 = \frac{1}{2}\frac{q_1^2}{C_s\varphi_m}, M_2 = \frac{1}{2}\frac{q_2^2}{\gamma_1 C_s \varphi_m}. \tag{4}$$

Both moments act on PM. It is considered that the moment of the elastic forces operating in the suspension of PM has the form $M_\varphi = k\varphi$, where k is torsional stiffness. The resultant moment of all active forces acting on the PM is defined as:

$$M_\varphi = k\varphi - \frac{1}{2}\frac{q_2^2 - \gamma_1 q_1^2}{\gamma_1 C_s \varphi_m}. \tag{5}$$

We consider that the moment of the frictional force M_ω is proportional to the rotational speed of the PM, so that $M_\omega = D_0\frac{d\varphi}{dt}$. Then, taking into account the relation between the directions of all moments of the forces and the direction of motion of PM considering Eq. (4), the PM equation of motion is then given by Eq. (6).

$$I_z\frac{d^2\varphi}{dt^2} + D_0\frac{d\varphi}{dt} + k\varphi - \frac{1}{2}\frac{q_2^2 - \gamma_1 q_1^2}{\gamma_1 C_s \varphi_m} = M(t), \tag{6}$$

where I_z is the PM's moment of inertia with respect to torsion axis and $M(t)$ is the moment of forces inertia or gravity regarded as the measuring signal of the sensor. Using Kirchhoff equations of the electric balance in the circuit of Fig. 2 and expressing the currents flowing through both capacitors in the form of $I_1 = \frac{dq_1}{dt}$ and $I_2 = \frac{dq_2}{dt}$, leads to the following two equations:

$$\frac{q_1}{C_1} + R\left(\frac{dq_1}{dt} - \frac{dq_2}{dt}\right) + R_1\frac{dq_1}{dt} = -V_1 + V,\ \frac{q_2}{C_2} + R\left(\frac{dq_2}{dt} - \frac{dq_1}{dt}\right) + R_2\frac{dq_2}{dt} = -V + V_2. \tag{7}$$

Particularly, in the case of static mode, when $\varphi = 0$ and $I_1 = I_2 = 0$, the charges in the capacitors based on Eqs (1) and (7) can be calculated by Eq. (8).

$$q_1 \equiv q_{1s} = C_s(V - V_1), q_2 \equiv q_{2s} = -C_s\gamma_1(V - V_2). \tag{8}$$

In the theory of differential equations, the system of Eqs (6) & (7) is known as “stiff” system. In our case, this leads during the numerical calculation to a growing number of steps and the calculation time grows unboundedly when decreasing values of resistors [17]. The solution of such systems requires special algorithms. Nowadays, these algorithms are well known, but in general case, the theory of finding precision solutions has not yet been developed.

For the purpose of simplicity in the analysis of sensor dynamics, Eqs. (6), (7) are linearized and reduced to a single linear differential equation of the fourth and the second order for the charge q_1 and the charge q_2 respectively. It is assumed that the charges consist of the permanent components q_s and variable components q_v as shown in Eq. (9).

$$q_1 = q_{1s} + q_{1v}, q_2 = q_{2s} + q_{2v}. \tag{9}$$

Because $I_1 = \frac{dq_{1v}}{dt}$ and $I_2 = \frac{dq_{2v}}{dt}$, by neglecting small terms, the following linear differential equation of the second order for total current $I = I_1 + I_2$ can be found (see Appendix A):

$$I_z\frac{d^2}{dt^2}I + p_5\frac{d}{dt}I + p_6 I = (\gamma_1 b_2 + b_1)\omega\frac{M_0}{\varphi_m}\sin(\omega t), \tag{10}$$

where

$$p_5 = D_0 + (\tau_1 + \tau_2\gamma_1)k - \frac{\gamma_1(\tau_2 b_1^2 + 2\tau b_1 b_2 + \tau_1 b_2^2)}{C_s\varphi_m^2}; p_6 = k - \frac{2b_1^2}{C_s\varphi_m^2} - P; P = \frac{\gamma_1 b_2^2 - b_1^2}{C_s\varphi_m^2} \tag{11}$$

The comparison of Eq. (10) with Eq. (6) shows the coefficient p_5 equivalent to the coefficient D_0, which considers the presence of viscous friction forces and the effect on resistors energy dissipation processes in the system; the coefficient p_6 is equivalent to the torsional stiffness k of