Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-14 (2015) 056–061

Design of H_∞ Controllers under Parametric Uncertainty and Power-Bounded External Disturbances

Vladimir N. Chestnov

V.A.Trapeznikov Institute of Control Science, Russian Academy of Sciences, Moscow, Russia, 117997 (Tel: +7495-334-7641; e-mail: vnchest@rambler.ru)

Abstract: We consider robust stabilization of linear MIMO systems, whose physical parameters are allowed to deviate from the nominal ones within known bounds, and the control plant is subjected to unknown power-bounded polyharmonic external disturbances (with unknown amplitudes and frequencies). The problem is to design a robustly stabilizing controller such that the prespecified errors for the controlled variables in steady state are guaranteed. The solution is based on the "loop-breaking technique" of the plant-controller system with respect to the physical parameters, e.g. Chestnov (1999); it reduces to the standard H_∞-optimization procedure by properly choosing the weighting matrix at the controlled variables. This approach is implemented numerically in the MATLAB-based Robust Control Toolbox (RCT).

Key words: linear multivariable systems, parametric uncertainty, bounded external disturbances, output feedback controller, H_∞-optimization, linear matrix inequalities.

1. INTRODUCTION

Robust stabilization of systems subjected to deviations of the parameters from their nominal values is the subject of numerous papers and books; e.g., see Ackermann (1993), Barmich (1994), Bhattacharyya et al. (1995), Zhou, et al. (1995), Zhou, et al. (1998). At the same time, as a rule, the majority of the papers consider the model parameters as the entries of the matrices in the state space equations or the coefficients of the transfer matrix of the plant. Generally speaking, these cannot be associated with the physical parameters of the plant, since both state-space equations and transfer matrix are secondary description tools for dynamic systems. Indeed, they are derived by converting the original equations of the dynamic system formulated from the fundamental laws of physics (mechanics, electro-dynamics, etc.). In this work, we deal with the parameters of such an intrinsic description, which have a transparent physical meaning, such as mass, moment of inertia, ohmic resistance, capacitance, inductance, etc. Moreover, as a rule, such a conversion of the original physical description leads to ``mixing'' and ``duplication'' of the varying parameters and hence, to a considerable complication of the problem and essentially conservative end-results.

In practice, real dynamic systems are affected by unknown external disturbances; in the mathematical control theory these disturbances are usually assumed to be bounded in certain norm, (e.g., see Chestnov (2011), Skogestad et al. (2007), Zhou et al. (1995), Zhou et al. (1998).

The approach developed here leans on the representation of dynamic systems in the so-called canonical (W, Λ, K)-form, (see Chestnov (1985), Chestnov (1995), Chestnov (1999)) such that, in the plant, the physical parameters (subjected to deviations from their nominal values) make up internal fictitious feedback loops in the form of the diagonal Λ matrix.

Unknown external disturbances are taken in the form of polyharmonic signals (with unknown amplitudes and frequencies), which are assumed to be power-bounded (i.e., for every coordinate of the disturbance, the sum of the squared amplitudes of each polyharmonic component is bounded). Similarly to Chestnov (2011), for the dynamic system we introduce the notion of the radius of the steady state with respect to the controlled variables. On top of robust stability, the desired controller must guarantee the specified (or, the minimal possible) value of the radius.

We show that such a problem reduces to the standard problem of rejection of exogenous disturbances in the framework of H_∞-approach (see Doyle et al. (1989)) by appropriately choosing the weighting matrix at the controlled variables of the plant.

This approach is implemented numerically as a code in the MATLAB-based Robust Control Toolbox (see Balas et al. (2010)). The idea of robust design using the (W, Λ, K)-representation was first proposed in Chestnov (1999), while the account for the accuracy is similar to the one in Aleksandrov, Chestnov (1998, a,b), Chestnov (2011). The design methodology is exemplified through the benchmark problem borrowed from in Haddad et al. (1993) and Farag et al. (2002).

2. STATEMENT OF THE PROBLEM

We consider a control plant described by the following equations in the physical variables:

$$\begin{cases} L_1(p)z_0(t) = L_2(p)u(t) + L_3(p)f(t) \\ y(t) = Nz_0(t) \end{cases} \tag{1}$$

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.09.433

where z_0 is the l-dimensional vector of the physical variables of the plant (velocity, acceleration, current, displacement, angle of rotation, etc.); u is the m-dimensional control input; y is the m_2-dimensional vector of observable (controllable) variables of the plant, f is the m_3-dimensional vector of external disturbances; N is a known numerical matrix of dimension $(m_2 \times l)$; $L_1(p)$, $L_2(p)$, $L_3(p)$ are polynomial matrices of dimensions $(l \times l)$, $(l \times m)$, $(l \times m_3)$, respectively, in the differentiation operator $p = d/dt$:

$$L_1(p) = \sum_{i=0}^{\alpha_1} L_1^{(i)} p^i , \qquad L_2(p) = \sum_{j=0}^{\alpha_2} L_2^{(j)} p^j ,$$

$$L_3(p) = \sum_{k=0}^{\alpha_3} L_3^{(k)} p^k , \qquad \alpha_2, \alpha_3 < \alpha_1$$

where $L_1^{(i)}, L_2^{(j)}, L_3^{(k)}$ are known real matrices of compatible dimensions.

In what follows, it is assumed that plant (1) is stabilizable and detectable, and its equations correspond to the original, ``least transformed'' description obtained from the fundamental laws of physics. In the sequel, the entries of the matrices will be referred to as the physical parameters of the plant. It is also assumed that n of the parameters entering (1) have nominal values $\lambda_1, \lambda_2, \ldots, \lambda_n$ and are allowed to vary in the given intervals:

$$\lambda_i + \Delta\lambda_i \in (\lambda_i^{\min}, \lambda_i^{\max}), \quad i = \overline{1, n}, \tag{2}$$

where $\Delta\lambda_i$ is the deviation of the i-th parameter from its nominal value.

Matrix elements $L_3(p)$ leave unaffected the stability of the closed-loop system, and therefore the deviations from the calculated not further considered here.

The coordinates of the vector external disturbance f are represented by bounded polyharmonic functions of the form

$$f_i(t) = \sum_{k=1}^{p_0} f_{ik} \sin(\omega_k t + \psi_{ik}), \quad i = \overline{1, m_3}. \tag{3}$$

Here, the amplitudes f_{ik}, the initial phases ψ_{ik} ($i = \overline{1, m_3}$, $k = \overline{1, p_0}$), and the frequencies ω_k ($k = \overline{1, p_0}$) of the harmonics are not known; however, the amplitudes satisfy the bounds

$$\sum_{k=1}^{p_0} f_{ik}^2 \le w_i^{*2} , \qquad i = \overline{1, m_3}, \tag{4}$$

where p_0 is a known number of harmonics, and the numbers w_i^* ($i = \overline{1, m_3}$) are given.

We define the steady-state errors with respect to the controlled variables by the following relation:

$$y_{i,st} = \lim_{t\to\infty} \sup |y_i(t)|, \quad i = \overline{1, m_2}. \tag{5}$$

Require that the output feedback controller provides the following conditions:

$$y_{i,st} \le y_i^*, \quad i = \overline{1, m_2}, \tag{6}$$

where $y_i^* > 0$, $i = \overline{1, m_2}$, are given numbers.

Clearly, there might exist no such controllers (see Aleksandrov et al. (1998, a)). Introduce the notion of the steady-state radius for the closed-loop system with respect to the controlled variables (e.g., see Aleksandrov, Chestnov (1998, b), Chestnov (2011)):

$$r_{st}^2 = \sum_{i=1}^{m_2} \left(\frac{y_{i,st}}{y_i^*} \right)^2, \tag{7}$$

which will be limited.

Problem 1: Synthesize a stabilizing output feedback controller

$$u(t) = K(p) y(t), \tag{8}$$

with the proper transfer matrix $K(p)$ such that the following holds:

(i) for given finite deviations of the parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$ from the nominal (2), the closed-loop system retains the asymptotic stability;

(ii) the steady-state radius for the controlled variables satisfy

$$r_{st}^2 \le \gamma^2, \tag{9}$$

where γ is a given (or minimal possible) number.

Obviously, for the problem to possess a solution, the assumption should be adopted on the retention of stabilizability and detectability of plant (1) under variations of the parameters within intervals (2).

To solve the problem, we follow the ``loop-breaking'' technique with respect to the varying parameters (e.g., see Chestnov (1999)) and represent the closed-loop equations (1), (8) in the diagonal canonical (W, Λ, K) – form with account for external disturbance (3).

3. THE CANONICAL (W, Λ, K)–FORM

The canonical (W, Λ, K)-representation of the closed-loop system with external disturbances has the form (see Chestnov (1985), Chestnov (1995), Chestnov (1999)):

$$\begin{aligned} \tilde{y} &= W_{11}\tilde{u} + W_{12}u + W_{13}f, \quad \tilde{u} = \Lambda\tilde{y} \\ y &= W_{21}\tilde{u} + W_{22}u + W_{23}f, \quad u = Ky \end{aligned} \tag{10}$$

where $W_{ij}(s)$ $(i=1,2,\ j=1\ldots3)$ are known transfer matrices which do not contain the varying parameters; u, y are, respectively, the physical input and output of plant (1); $\tilde{u}$, $\tilde{y}$ are n - dimensional fictitious input and output of the plant; $\Lambda = \mathrm{diag}[\lambda_1, \lambda_2, \ldots, \lambda_n]$ is the diagonal matrix of the parameters subjected to deviations around the nominal; K is the desired transfer matrix of controller (8).

The block-diagram associated with representation (10) is depicted in Figure 1.