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Variable Reference Model for Model Reference Adaptive Control System

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Abstract: In a conventional model reference adaptive control system, a reference model which has desired response characteristics is designed in advance. However, there is a fear that a control output cannot follow a reference model output under the model uncertainty. In this paper, we propose an adaptive control scheme with variable reference model based on the partial model matching method and show the effectiveness by a numerical simulation.

Keywords: adaptive control, variable reference model.

1. INTRODUCTION

Model reference adaptive control system (MRACS) is one of the main approaches to adaptive control. The basic structure of a MRACS is shown in Fig. 1. The reference model is chosen to generate the desired trajectory $y_M(t)$ that the plant output $y(t)$ has to follow. The tracking error $\varepsilon(t)$ represents the deviation of the plant output from the desired trajectry. The closed-loop plant is made up of an ordinary feedback control law that contains the plant and a controller and an adjustment mechanism that generates the controller parameter estimates θ on-line.

However, this reference model should also be essentially (re-)designed according to characteristics of a controlled object. The purpose of this paper is to design not only the controller and parameter adjustment mechanism but also reference model in Fig. 2 so that all signals in the closed-loop plant are bounded and the plant output $y(t)$ tracks $y_M(t)$ as close as possible.

This paper deals with the position control of DC servomotor and is organized as follows; A mathematical model of the DC servomotor is presented in section 2. Reference model design algorithms are developed in section 3. Adaptive PD control is presented in section 4 and the effectiveness of the proposed method is confirmed by numerical example in section 5.

2. DC SERVOMOTER

2.1 Mathematical Model

In this paper, we consider an armature controlled DC servomotor in Fig. 3. The utility of this DC servomotor is to provide a torque via armature current to rotate the load. The mathematical model of DC servomotor in common estabilishes the relationship betewen the output (speed or position) and the input (voltage, current). The voltage induced between the output of DC servomotor is given by

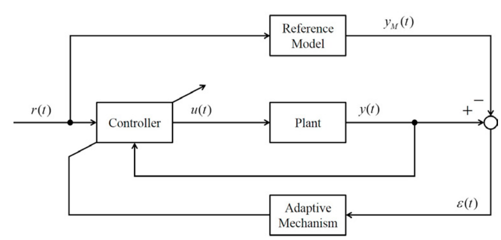

Fig. 1. Structure of conventional MRACS

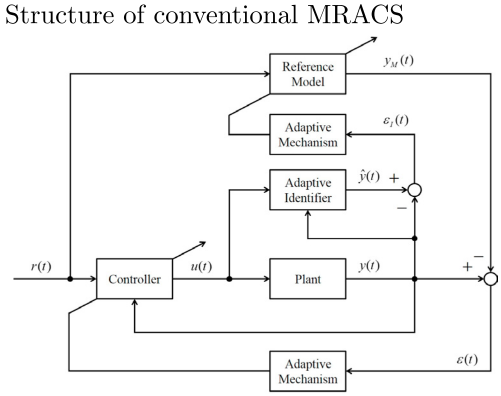

Fig. 2. MRACS with variable reference model

$$v_e(t) = k_e \omega(t) \tag{1}$$

where k_e, and $\omega(t)$ represent respectively the voltage constant and the motor angular velocity. Let $i(t)$ be the armature current and $u(t)$ be the voltage applied to the armature, the electrical differential equation for a DC servomotor can be written as:

$$u(t) = L\frac{di(t)}{dt} + Ri(t) + v_e(t) \tag{2}$$

where R and L represent respectively the resistance and inductance of the armature. The torque $\tau(t)$ developped by the DC servomotor is proportional to the current in the winding.

$$\tau(t) = k_\tau i(t) \tag{3}$$

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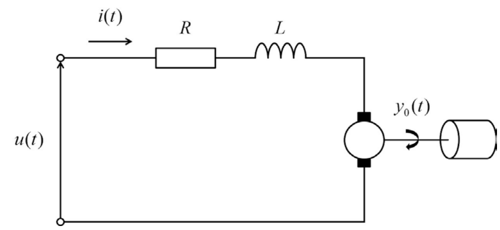

Fig. 3. Circuit diagram of DC servomoter

where k_τ is the torque constant. The DC servomotor torque can also be written as:

$$\tau(t) = J\frac{d\omega(t)}{dt} + B\omega(t) \tag{4}$$

where J and B are respectively the inertia moment and the friction coefficient of the motor and load. Using (1)-(4), we obtain the following transfer function of the DC servomotor.

$$G(s) = \frac{b_0}{a_1 s + a_2 s^2 + a_3 s^3} \tag{5}$$

where $a_1 = RB + k_e k_\tau, a_2 = RJ + LB, a_3 = LJ, b_0 = k_\tau$.

2.2 Problem Statement

Let us consider Single-Input/Single-Output plant:

$$y_0(t) = G(s)u(t) \tag{6}$$

$$y(t) = y_0(t) + d(t) \tag{7}$$

and reference model:

$$y_M(t) = G_M(s)r(t) \tag{8}$$

where $d(t)$ is a bounded disturbance and $r(t)$ is a bounded piecewice continuous signal. The control objective is to design $u(t)$ such that $y(t)$ asymptotically tracks $y_M(t)$ with all generated signals remaining bounded.

3. VARIABLE REFERENCE MODEL

3.1 Partial Model Matching Method

Partial model matching method proposed by Kitamori is developed which enables one to design control systems of various types in the situation that the dynamic characteristics of the controlled process are not known perfectly. The control system to be designed is to satisfy the following specifications: (a) It has zero steady-state error. (b) It has adequate damping characteristics. (c) (a) and (b) satisfied, it has a shortest rise-time. For the design, a sequence of parameters is found suitable to represent the dynamics of an element or a system. The sequence is equivalent to that of moments of the impulse response in the sense of I. F. S. (independency from the successors) which plays a fundamental role in the development. The specifications above are also re-organized into a sequence of conditions on the parameters of the control system to be designed. When we define the denominator-expanded form:

$$G(s) = 1/H(s) \tag{9}$$

$$H(s) = h_1 s + h_2 s^2 + h_3 s^3 \tag{10}$$

$$G_M(s) = 1/H_M(s) \tag{11}$$

$$H_M(s) = 1 + \sigma s + \alpha_2(\sigma s)^2 + \alpha_3(\sigma s)^3 \tag{12}$$

where $h_1 = a_1/b_0, h_2 = a_2/b_0, h_3 = a_3/b_0$, the time-scaling parameter σ and control gains K_P, K_D are calculated as

$$\sigma = \frac{\alpha_2 h_3}{\alpha_3 h_2} \tag{13}$$

$$K_P = \frac{h_2}{\alpha_2 \sigma^2} \tag{14}$$

$$K_D = K_P\sigma - h_1 \tag{15}$$

based upon partial model matching method.

3.2 Adaptive Identification

To design a reference model, we have to calculate σ. However, right side of (13) includes parameters of controlled object. Therefore, we first identify system parameters. The control output $y(t)$ is parametrically transformed using positive number λ.

$$y(t) = \phi^{\mathrm{T}}\zeta_I(t) + z(t) \tag{16}$$

$$\phi = [\phi_1 \ \phi_2 \ \phi_3 \ \phi_4]^{\mathrm{T}} \tag{17}$$

$$\zeta_I(t) = \left[\frac{1}{(s+\lambda)^3}u(t) \ \frac{1}{s+\lambda}y(t) \ \frac{1}{(s+\lambda)^2}y(t) \ \frac{1}{(s+\lambda)^3}y(t)\right]^{\mathrm{T}} \tag{18}$$

$$z(t) = \frac{a_1' s + a_2' s^2 + s^3}{(s+\lambda)^3}d(t) \tag{19}$$

where $\phi_1 = b_0, \phi_2 = 3\lambda - a_2', \phi_3 = -3\lambda^2 + 2\lambda a_2' - a_1', \phi_4 = \lambda^3 - \lambda^2 a_2' + \lambda a_1', a_1' = a_1/a_3, a_2' = a_2/a_3, b_0' = b_0/a_3$. We define the estimate value of ϕ as $\hat{\phi}(t)$ and the identification model $\hat{y}(t)$ and estimation error $\varepsilon_I(t)$ are expressed as

$$\hat{y}(t) = \hat{\phi}^{\mathrm{T}}(t)\zeta_I(t) \tag{20}$$

$$\varepsilon_I(t) = \hat{y}(t) - y(t) \tag{21}$$

$$= \tilde{\phi}^{\mathrm{T}}(t)\zeta_I(t) - z(t) \tag{22}$$

The estimation is implemented by following dead-zone and adaptive law.

$$\dot{\hat{\phi}}(t) = -\Gamma_I\zeta_I(t)D(\varepsilon_I(t)) \tag{23}$$

$$D(\varepsilon_I(t)) = \begin{cases} \varepsilon_I(t) - d_{0_I} & \varepsilon_{I_N}(t) > d_{0I} \\ 0 & |\varepsilon_I(t)| \le d_{0I} \\ \varepsilon_I(t) + d_{0_I} & \varepsilon_{I_N}(t) < -d_{0I} \end{cases} \tag{24}$$

where Γ_I is positive definite symmetric matrix and $d_{0I} \ge \sup_t |z(t)|$. From this estimation, we can get $\hat{\sigma}$ as:

$$\hat{\sigma} = \frac{\alpha_2}{\alpha_3(3\lambda - \hat{\phi}_2)} \tag{25}$$