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Delay-Interval-Dependent Stability Criterion for Linear Systems With Time-Varying State Delay

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Abstract: This paper considers the problem of stability analysis of linear systems with a time-varying state delay. A new delay-interval-dependent stability criterion is derived without involving any direct approximation of the integral terms that involve uncertain limit of integration in the time-derivative of the Lyapunov-Krasovskii functional. This development leads to a less conservative LMI criterion as seen through numerical examples.

Keywords: Time-Delay Systems, Stability Analysis, Lyapunov-Krasovskii Functional and LMI.

1. INTRODUCTION

Consider a linear system with time-varying delay as

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)), \tag{1}$$

where $x(t) \in \Re^n$ is the state of the system; $\tau(t)$ is a time-varying state-delay satisfying

$$0 \leq \tau_1 \leq \tau(t) \leq \tau_2, \quad \bar{\tau} = \tau_2 - \tau_1, \quad \dot{\tau}(t) \leq \mu; \tag{2}$$

In this paper, we consider the problem of deriving a delay-dependent stability criterion for such systems based on simple Lyapunov-Krasovskii (LK) functional. Such a criterion may be used to compute the tolerable delay bound τ_2 for given τ_1 or vice versa.

Delay-dependent stability of time-delay systems has been widely investigated in recent years based on LK approach (see Y. S. Moon (2001), E. Fridman (2002), E. Fridman (2003), H. J. Gao (2003), Y. He (2004), M. Wu (2004) for $\tau_1 = 0$ and Y. He (2007), X. Jiang (2005), Shao (2008), Shao (2009) for $\tau_1 \geq 0$). These approaches lead to stability criterion in terms of Linear Matrix Inequalities (LMIs). A transformed model of (1) has been used in Y. S. Moon (2001) to analyze stability. But such transformations introduce additional dynamics in the transformed model leading to conservative results K. Gu (2000). Descriptor model transformation has been used in E. Fridman (2002, 2003) by first describing the system into descriptor form and then incorporating $\dot{x}(t)$ in the bounding of an integral term leading to less conservative results. The same integral term has more tightly been approximated in Y. He (2004); X. Jiang (2005) by using some free weighting matrices. However, they neglected a component of the integral term which has appropriately been considered in Y. He (2007) to obtain less conservative results. Recently in Kim (2011) an improved delay upper bound result for constant as well as very low delay variation rate (μ) cases satisfying the condition $\tau(t) \in [0,\ \tau_2]$ has been obtained by adopting quadratic terms multiplied by higher order scalar functions. Formulation of delay-range-dependent stability analysis with or without delay-partitioning utilizing this method will lead to the use of more matrix variables and more bounding inequalities as delay ranges are sub-divided here, in such a situation the adoption of proposed method can limit to greater extent the rise in number of decision variables.

In concern to the bounding of the integral term discussed above, one employs an approximation in free variable matrix approach X. M. Zhang (2005) as:

$$-\int_{t-\beta}^{t-\alpha} \dot{x}^T(\theta) R \dot{x}(\theta) d\theta \leq \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}^T \left\{ \begin{bmatrix} M + M^T & -M + N^T \\ * & -N - N^T \end{bmatrix} + \gamma \begin{bmatrix} M \\ N \end{bmatrix} R^{-1} \begin{bmatrix} M \\ N \end{bmatrix}^T \right\} \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}, \tag{3}$$

where $*$ represents symmetric components, $R = R^T > 0$, $\beta > \alpha \geq 0$, $\gamma = \beta - \alpha > 0$ and M, N are free weighting matrices of appropriate dimension. In Shao (2008), it has been shown that use of such free weighting matrices may impose constraint on the resulting stability criterion

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and obtain less conservative results by using Jensens's inequality of Gu (2000) (not having any free matrix) given by

$$-\int_{t-\beta}^{t-\alpha} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta \leq \gamma^{-1}\begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}^T \begin{bmatrix} -R & R \\ * & -R \end{bmatrix} \begin{bmatrix} x(t-\alpha) \\ x(t-\beta) \end{bmatrix}. \quad (4)$$

Many attempts have been made to deduce equivalency of several stability criteria based on either (3) or (4), e.g. see S. Xu (2007); Shao (2008). Explicit relation between (3) and (4) can be established following the equivalency results in Briat (2011). In this regard, note that, the first term in the RHS of (3) may be represented as:

$$\begin{bmatrix} M \\ N \end{bmatrix} \begin{bmatrix} I \\ -I \end{bmatrix}^T + \begin{bmatrix} I \\ -I \end{bmatrix} \begin{bmatrix} M \\ N \end{bmatrix}^T. \quad (5)$$

Now, it is easy to see that (3) and (4) are equivalent in view of Theorem 4.1 of Briat (2011). Moreover, the RHS of (3) is minimum when

$$M = M^T = -N = -N^T = -\gamma^{-1}R, \quad (6)$$

and for such a choice (3) becomes (4).

From the above, it seems that use of (4) has the advantage that it does not involve additional free variables besides being equivalent to (3). However, if γ is uncertain and required to be approximated with its lower or upper bound then use of (3) would be beneficial since the choice (6) can not be met with an approximated γ. The uncertainty in γ arises when certain limit of integral is a function of time (i.e., $\tau(t)$ a time varying delay in this case). In such a situation approximating the integral term (as in (4) proves to be conservative as large gap exists between right hand side (RHS) and left hand side (LHS), so reduction of the gap is the main contribution of this paper as it eventually gets reflected in the improved estimate of delay bound. Moreover, the RHS of (3) is affine on the parameter γ, which is beneficial in formulating convex combinations, specifically when the stability criterion is sought in terms of LMIs. Based on this observation, we derive a delay-range dependent stability criterion by suitably using both the inequalities. Recently, an attempt has been made to use this affine feature by introducing a new type of inequality in P. Park (2011), but it still involves approximation of uncertain delay term in the derivation.

To this end, for a chosen LK functional, the ways of reducing conservativeness are in (i) using tighter bounding of the integral term and (ii) avoiding neglecting any term in the derivative of the LK functional. For the first one, it is already shown that the inequalities (3) and (4) are equivalent excepting only use of former one introduces additional free variables. In addition, it may also be possible to reduce the conservativeness in (3) or (4) by discretization of the interval Briat (2011), however, this aspect is not considered in this paper. For the second purpose, if γ is uncertain then using (3), due to its affine dependence on the uncertain delay term, is advantageous to express the resulting criterion in terms of LMIs.

In this paper, we derive a stability criterion for system (1) by suitably using both (3) and (4). The former one is used only when γ is uncertain. It is seen that such an use of these two bounding inequalities leads to a convenient LMI criterion that takes benefit of not using free variables (wherever possible, by the use of (4)) as well as allow us to avoid omitting uncertain delay terms (by using (3)). It is also shown by deriving a stability criterion only using (4) as a special case that, due to the approximations involved, yields conservative results.

2. STABILITY ANALYSIS

The following theorem presents the stability criterion for system (1) in terms of LMIs.

Theorem 1. System (1) is asymptotically stable if there exist symmetric matrices $P > 0$, $Q_i > 0$, $i = 1, \ldots, 4$, $R_j > 0$, and arbitrary matrices M_j, N_j, satisfying the following LMIs for $j = 1, 2$:

$$\begin{bmatrix} \Theta & \Phi_j \\ * & -R_2 \end{bmatrix} < 0, \quad (7)$$

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & 0 \\ * & \Theta_{22} & \Theta_{23} & 0 \\ * & * & \Theta_{33} & \Theta_{34} \\ * & * & * & \Theta_{44} \end{bmatrix}, \Phi_1 = \begin{bmatrix} 0 \\ M_1 \\ N_1 \\ 0 \end{bmatrix},$$

$$\Phi_2 = \begin{bmatrix} 0 \\ 0 \\ M_2 \\ N_2 \end{bmatrix},$$

$$\begin{aligned}
\Theta_{11} &= PA + A^TP + \sum_{i=1}^{3} Q_i + A^T(\tau_1^2R_1 + R_2)A - R_1, \\
\Theta_{12} &= R_1, \Theta_{13} = PA_d + A^T(\tau_1^2R_1 + R_2)A_d, \\
\Theta_{22} &= Q_4 - Q_1 - R_1 + \bar{\tau}^{-1}(M_1 + M_1^T), \\
\Theta_{23} &= \bar{\tau}^{-1}(-M_1 + N_1^T), \\
\Theta_{33} &= -(1-\mu)(Q_3 + Q_4) + A_d^T(\tau_1^2R_1 + R_2)A_d, \\
&\quad + \bar{\tau}^{-1}(M_2 + M_2^T - N_1 - N_1^T), \\
\Theta_{34} &= \bar{\tau}^{-1}(-M_2 + N_2^T), \Theta_{44} = -Q_2 - \bar{\tau}^{-1}(N_2 + N_2^T).
\end{aligned}$$

Proof. Define a Lyapunov-Krasovskii functional as

$$\begin{aligned}
V(t) &= x^T(t)Px(t) + \sum_{i=1}^{2}\int_{t-\tau_i}^{t} x^T(\theta)Q_ix(\theta)d\theta \\
&+ \int_{t-\tau(t)}^{t-\tau_1} x^T(\theta)Q_4x(\theta)d\theta + \tau_1\int_{t-\tau_1}^{t}\int_{\theta}^{t} \dot{x}^T(\phi)R_1\dot{x}(\phi)d\phi d\theta \\
&+ \bar{\tau}^{-1}\int_{t-\tau_2}^{t-\tau_1}\int_{\theta}^{t} \dot{x}^T(\phi)R_2\dot{x}(\phi)d\phi d\theta + \int_{t-\tau(t)}^{t} x^T(\theta)Q_3x(\theta)d\theta
\end{aligned} \quad (8)$$

Time-derivative of (8) along the state trajectory of (1) is

$$\dot{V}(t) = 2x^T(t)PAx(t) + 2x^T(t)PA_dx(t-\tau(t))$$