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Robust Fault-Tolerant Control based on a Functional Observer for Linear Descriptor Systems

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Abstract: A novel robust fault-tolerant control design for linear descriptor systems is proposed in this paper, with fault tolerant control action achieved via a normal state estimate feedback derived using H_∞ optimization and fault compensation. A robust functional (reduced order) observer approach is proposed to estimate the controller directly. Necessary and sufficient conditions are given to ensure the asymptotic stability of the functional observer. H_∞ optimization with LMI regional pole placement is used to design the functional observer so that satisfactory estimation and closed-loop system transient performances are achieved, subject to bounded faults and disturbance. Then, a step-by-step fault-tolerant control design procedure for obtaining both the controller and observer design parameters is given. Finally, the effectiveness of the proposed design is illustrated using a numerical example.

Keywords: Robust Fault-Tolerant Control, Functional Observer, H_∞ Optimization, Pole Placement, Linear Descriptor Systems

1. INTRODUCTION

Fault-tolerant control (FTC) has been regarded as one of the important research fields in the control community, and has inspired significant research on both theory and application aspects, see Patton (1997), Blanke et al. (2006), Zhang and Jiang (2008), Ding (2009), Sami and Patton (2013), Feng and Patton (2014) and references therein. The FTC designs in the literature are mainly based on either fault diagnosis (Blanke et al. (2006), Zhang and Jiang (2008), Ding (2009), and etc.) or fault estimation (Gao and Ding (2007), Sami and Patton (2013), Zhang et al. (2013), and Feng and Patton (2014)).

Descriptor systems are important in a number of fields of engineering, such as power systems, electrical networks, robotics, aerospace systems, chemical processes, social economic systems, biological systems, time-series analysis, etc., and have been widely researched for the past 30 years, see the books Dai (1989), Duan (2010), and references therein.

However, for FTC, only a few works have been published. In Gao and Ding (2007) and Shi and Patton (2015), robust reconfigurable control designs were presented based on the estimated states and faults through an extended state observer, while in Marx et al. (2004), a co-prime factorization-based approach was proposed to obtain all the robust fault-tolerant Youla controllers for the systems subject to faults and disturbance. Although the previous designs can achieve good system performance despite the presence of bounded faults and disturbance, the observers proposed are of high order, leading to complex design for high dimension systems. In this respect, a novel robust FTC design based on a functional (reduced order) observer is of interest here.

A functional observer aims to design a reduced or even minimal order observer to estimate a linear function of the states for a system. It was first presented in Luenberger (1966), and further extended to estimate a preferred linear function of the states with unknown input, see Hou et al. (1999), Xiong and Saif (2003), Deng and Li (2004), Trinh and Fernando (2012), and the references therein. Attention has also been paid to the functional observer design for descriptor systems (Fernando and Trinh (2007), Zhou et al. (2008), and Darouach (2012)) to estimate a linear function of the states and external disturbances. Recently, functional observers have been proposed to estimate the controller signal (as a linear function of the states) for the cases (a) without disturbance (Ha et al. (2003)) and (b) with disturbance (Ezzine et al. (2012)), respectively. In this paper, the functional observer is used to estimate an FTC system controller for linear descriptor systems subject to faults and disturbance; this is different from the observers proposed in the literature studies. The optimization designs are achieved using an LMI formulation.

However, it is often the case that no guarantee of good system transient performance exists by only using H_∞ optimization. One reasonably effective approach to ensure a satisfactory performance is to use an LMI regional pole placement design strategy (Chilali et al. (1999), Marx et al. (2003), Patton et al. (2012), and Shi and Patton (2015)).

Hence, a novel robust FTC design is proposed here with fault compensation based on a functional observer also designed via LMI regional pole placement. Compared with existing studies, the main contributions cover two aspects: (1) a robust functional observer with reduced order is

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proposed to estimate the fault-tolerant controller (a linear function of the states and faults); and (2) techniques of LMI regional pole placement and H_∞ control are used together to guarantee satisfactory transient performance of the observer and closed-loop system.

The paper is organized as follows. Section 2 formulates the problem and gives some preliminaries. Next, Section 3 proposes the functional observer and controller designs, respectively. Then, an illustrative example is provided in Section 4. Finally, Section 5 concludes the paper.

Notation: The symbol † represents the pseudo inverse, $\otimes$ represents the Kronecker product, $Herm(Q) = Q + Q^\top$, and $\star$ represents the symmetric part of a matrix.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the linear descriptor system:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) + Ff(t) + Dd(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input vector, $y \in R^p$ is the system output, $f \in R^q$ denotes the unknown fault, and $d \in R^l$ represents the disturbance. $E \in R^{n\times n}$, $A \in R^{n\times n}$, $B \in R^{n\times m}$, $F \in R^{n\times q}$, $D \in R^{n\times l}$ and $C \in R^{p\times n}$ are known constant matrices. $rank(E) = r \leq n$. The time index is omitted for simplicity in the following sections.

In this study, the following Assumptions are made:

Assumption 2.1. The pair (E, A, C) is observable, (E, A, B) is controllable, and (E, A) is regular.

Assumption 2.2. $rank(B,\ F) = rank(B)$.

Assumption 2.3. The fault f belongs to $\mathcal{L}_2[0, \infty)$ and it is slowly varying, i.e., $\dot{f} \approx 0$.

The aim of this paper is to design a robust FTC based on a functional observer for (1) such that its closed-loop system is stable in the presence of bounded faults and disturbance. Consider the following augmented form of (1):

$$\begin{aligned} \bar{E}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u + \bar{D}d \\ y &= \bar{C}\bar{x} \end{aligned} \tag{2}$$

where

$$\bar{x} = \begin{bmatrix} x \\ f \end{bmatrix},\quad \bar{E} = \begin{bmatrix} E & 0 \\ 0 & I_q \end{bmatrix},\quad \bar{A} = \begin{bmatrix} A & F \\ 0 & 0 \end{bmatrix},\quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},$$
$$\bar{D} = \begin{bmatrix} D \\ 0 \end{bmatrix},\quad \bar{C} = [C\ 0].$$

The following Lemmas and Definitions are used in the design procedure.

Lemma 2.1. (Uezato and Ikeda (1999)) Consider the following linear descriptor system:

$$\begin{aligned} E\dot{x} &= Ax + Rd \\ z &= Cx. \end{aligned} \tag{3}$$

The pair (E, A) is admissible and $\|G_{zd}\|_\infty < \gamma$ for some positive scalar γ , if and only if there exist a positive definite matrix $W \in R^{n\times n}$ and a matrix $S \in R^{(n-r)\times(n-r)}$ such that:

$$\begin{bmatrix} \Pi_1 & R & \Pi_2 \\ \star & -\gamma I & 0 \\ \star & \star & -\gamma I \end{bmatrix} < 0$$

where $\Pi_1 = Herm\left(A(WE^\top + USV^\top)\right)$, $\Pi_2 = (WE^\top + USV^\top)^\top C^\top$, the matrices U and V are of full-column rank and contain the bases of $Ker(E)$ and $Ker(E^\top)$, respectively.

Based on the work in Chilali et al. (1999), the following definitions can be given.

Definition 2.1. (LMI region) A subset $\mathcal{D}$ of the complex plane is called an LMI region if there exists a symmetric matrix $\alpha = [\alpha_{kl}] \in R^{q_l\times q_l}$, and a matrix $\beta = [\beta_{kl}] \in R^{q_l\times q_l}$ such that:

$$\mathcal{D} := z \in \mathcal{C} : f_{\mathcal{D}}(z) < 0$$

where the characteristic function $f_{\mathcal{D}}(z)$ is defined by:

$$f_{\mathcal{D}}(z) := \alpha + z\beta + \bar{z}\beta^\top = [\alpha_{kl} + \beta_{kl}z + \beta_{lk}\bar{z}]_{1\leq k,l\geq q_l}.$$

Definition 2.2. ($\mathcal{D}$ stability) A descriptor system is said to be $\mathcal{D}$ stable if all its finite eigenvalues belong to an LMI region $\mathcal{D}$.

Lemma 2.2. (Quadratic $\mathcal{D}$-admissibility) (Shi and Patton (2015)) A descriptor system is said to be quadratically $\mathcal{D}$-admissible if there exist a positive definite matrix $W \in R^{n\times n}$ and a matrix $S \in R^{(n-r)\times(n-r)}$ such that:

$$\alpha \otimes (EWE^\top) + Herm\left(\beta \otimes (AWE^\top) + I_{q_1} \otimes (AUSV^\top)\right) < 0$$

where U and V full-column rank matrices that contain the bases of $Ker(E)$ and $Ker(E^\top)$, respectively, and I_{q_1} denotes an identity matrix.

3. MAIN RESULT

3.1 Functional observer design

Assume that a suitable FTC controller for (2) is $u = K\bar{x}$ with $K \in R^{m\times(n+q)}$. The following observer is given to estimate $z = K\bar{x}$ with $z \in R^m$:

$$\begin{aligned} \dot{\xi} &= N\xi + Jy + Ru \\ \hat{z} &= \xi + My \end{aligned} \tag{4}$$

where $\xi, \hat{z} \in R^m$ denote the observer state and the estimate of z, respectively. Matrices $N \in R^{m\times m}$, $J \in R^{m\times p}$, $R \in R^{m\times m}$, and $M \in R^{m\times p}$ are to be determined such that $\lim_{t\to\infty}(\hat{z} - z) = 0$.

Assume that $rank \begin{bmatrix} \bar{C} \\ K \end{bmatrix} = p + m \leq n + q$. Let $T \in R^{m\times(n+q)}$ be a full-row rank matrix. Define the estimation errors as:

$$e = \hat{z} - z,\quad \varepsilon = \xi - T\bar{E}\bar{x},$$

it then follows that: