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H-Infinity Control of MEMS Gyroscope Using T-S Fuzzy Model

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Abstract: A multi-input multi-output (MIMO) Takagi–Sugeno (T–S) fuzzy model is proposed to represent the nonlinear model of micro-electro mechanical systems (MEMS) gyroscope. A Lyapunov-based H- infinity control strategy is derived to attenuate the influence of the various external disturbances within the desired region. The H-infinity control technique has been combined with an adaptive control method to achieve the desired attenuation of disturbance due to the parameter uncertainties and external disturbances in the MEMS gyroscopes. For the purpose of comparison, the designed controller is also implemented on the nonlinear model of the MEMS gyroscope. Numerical simulations are investigated to verify the effectiveness of the proposed control scheme on the T-S model and the nonlinear model.

Keywords: H-infinity control, LMI inequality, T-S Fuzzy Model, nonlinear model.

1. INTRODUCTION

MEMS gyroscopes have become the most growing micro-sensors for measuring angular velocity in recent years due to their compact size, low cost and high sensitivity. The performance of the MEMS gyroscopes is often deteriorated by the effects of time-varying parameters, quadrature errors, external disturbances, cross stiffness and damping effects generating from fabrication imperfections. However, advanced control algorithm can be utilized to control the MEMS gyroscopes and improve their performance. In the last few years, various advance control approaches have been proposed to control the MEMS gyroscope. presented an adaptive controller for a MEMS gyroscope was presented to drive both axes of vibration and control the entire operation of the gyroscope (Park et al. 2007). A novel concept for an adaptively controlled triaxial angular velocity sensor device was proposed (John et al. 2007). A phase-domain approach to design the controller for the gyroscope was introduced (Sun et al. 2009) . a sliding mode control for a MEMS gyroscope system was developed (Batur et al. 2006). Adaptive sliding mode control approach was developed to control the MEMS gyroscope (Fei et al. 2009). An adaptive fuzzy sliding controller and adaptive fuzzy compensator were proposed for MEMS triaxial gyroscope (Fei 2010, Fei. et al. 2012). An adaptive fuzzy sliding controller using feedback linearization was proposed for the gyroscope with unknown system nonlinearities (Fei et al. 2013).

The system nonlinearities in MEMS gyroscope model was described (Asokanthan et al. 2009) . Use the fuzzy implications and the fuzzy reasoning methods (Takagi et al. 1985) , a real nonlinear plant model could be constructed by local linear models. a robust adaptive controller design was developed for a class of uncertain nonlinear systems using online T–S fuzzy-neural modeling approach (Chien et al. 2011). A T–S model based adaptive fuzzy controller was designed using online parameter estimation (Park et al.2004) . Advanced control algorithm is utilized to compensate the error caused by the uncertainties in the real nonlinear system including external disturbance, and the transformation from the real system to the T-S fuzzy model. Intelligent control approaches such as fuzzy control do not require mathematical models and can approximate smooth nonlinear systems, while the fuzzy approximator is unavailable to approximate the reconstruction error with absolutely accuracy. H-infinity controller is more robust and less sensitive to the model uncertainties and external disturbances. A controller which can guarantee the convergence of control state and uniformly bounded while maintaining all the signals involved stable was proposed (Han 2014). The quadratic stability conditions and H-infinity controller for T-S fuzzy systems were discussed (Liu 2003). Other H-infinity controllers for stochastic T-S fuzzy systems have been investigated (Chang et al.2014, Kim 2014, Gao et al. 2014). However, systematic stability analysis and controller design of the H infinity controller using T-S fuzzy model for the MEMS gyroscope have not been found in the literature. Thus, H infinity control using T-S fuzzy model is utilized to approximate the nonlinear system, compensate reconstruction error and external disturbances, thereby improving tracking and compensation performance.

In this paper, a Lyapunov-based H-infinity control strategy is applied to the tracking control of MEMS gyroscope using T-S fuzzy model. The proposed T–S fuzzy model is suitable for complex nonlinear systems, since the control efficiency depends on the accuracy of T–S model, which can use few rules to describe a complex nonlinear system with a specific solving method. The proposed control strategy has the following characteristics and contributions:

1).T-S modeling method provides a possibility for developing a systematic analysis and design method for complex nonlinear control systems, thus improving the tracking and compensation performance. The MIMO T-S fuzzy model with input disturbance of the MEMS gyroscope

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is established based on the non-dimensional equation of MEMS gyroscope. According to the Parallel Distributed Compensation (PDC) principle, the local state controllers are designed for each linear sub-model.

2). The advantage of the proposed H-infinity controller over other controllers is that it can simplify the control algorithm and improve the real-time of the control system. A H-infinity controller with a feedforward controller and a feedback controller is proposed in the control of the MEMS gyroscope. Based on the Lyapunov theory and the H-infinity theory, a LMI inequality is designed. The proposed H-infinity controller can guarantee the asymptotic stability of the closed loop system and ensure the impact of external disturbance to the control system within the desired indicators in the presence of different external disturbances.

2. DYNAMICS OF MEMS GYROSCOPE

The dynamics of a z-axis MEMS gyroscope showing in Fig. 1 through nondimensional transformation is discussed. Assume that the gyroscope is moving with a constant linear speed; the gyroscope is rotating at a constant angular velocity; the gyroscope undergoes rotations along z axis. The nonlinear motion equation can be derived as:

$$m\ddot{x} + d_{xx}\dot{x} + \left(d_{xy} - 2m\Omega_z^*\right)\dot{y} + \left(k_{xx} - m\Omega_z^{*2}\right)x + k_{xy}y + k_{x^3}x^3 = u_x^*$$
$$m\ddot{y} + d_{yy}\dot{y} + \left(d_{xy} + 2m\Omega_z^*\right)\dot{x} + \left(k_{yy} - m\Omega_z^{*2}\right)y + k_{xy}x + k_{y^3}y^3 = u_y^* \quad (1)$$

where x, y represents the system generalized coordinates, m is the mass of proof mass. Fabrication imperfections contribute mainly to the asymmetric spring term d_{xy}, and asymmetric damping terms k_{xy}; d_{xx}, d_{yy} are damping terms; k_{xx}, k_{yy} are linear spring terms; k_{x^3}, k_{y^3} are nonlinear spring terms; Ω_z^* is the input angular velocity; u_x^*, u_y^* are the control forces.

Dividing the equation by the reference mass, and because of the non-dimensional time $t^* = \omega_0 t$, dividing both sides of equation by reference frequency ω_0^2 and reference length q_0 and rewriting in vector form result in

$$\frac{\ddot{q}_1^*}{q_0} + \frac{D^*}{m\omega_0}\frac{\dot{q}_1^*}{q_0} + 2\frac{S^*}{\omega_0}\frac{\dot{q}_1^*}{q_0} - \frac{\Omega_z^{*2}}{m\omega_0}\frac{q_1^*}{q_0} + \frac{K_1^*}{m\omega_0^2}\frac{q_1^*}{q_0} + \frac{K_3^*}{m\omega_0^2}\frac{q_1^{*3}}{q_0} = \frac{u^*}{m\omega_0^2 q_0} \quad (2)$$

where

$$q_1^* = \begin{bmatrix} x \\ y \end{bmatrix}, u^* = \begin{bmatrix} u_x^* \\ u_y^* \end{bmatrix}, D^* = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}, S^* = \begin{bmatrix} 0 & -\Omega_z^* \\ \Omega_z^* & 0 \end{bmatrix},$$
$$K_1^* = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}, K_3^* = \begin{bmatrix} k_{x^3} & 0 \\ 0 & k_{y^3} \end{bmatrix}.$$

Define a set of new parameters as follows:

$$q_1 = \frac{q_1^*}{q_0}, u = \frac{u_1^*}{m\omega_0^2 q_0}, \Omega_z = \frac{\Omega_z^*}{\omega_0},$$
$$D = \frac{D^*}{m\omega_0}, K_1 = \frac{K_1^*}{m\omega_0^2}, K_3 = \frac{K_3^* q_0^2}{m\omega_0^2}.$$

The final form of the non-dimensional motion equation for the z-axis gyroscope is

$$\ddot{q}_1 = (2S - D)\dot{q}_1 + \left(\Omega_z^2 - K_1\right)q_1 - K_3 q_1^3 + u \quad (3)$$

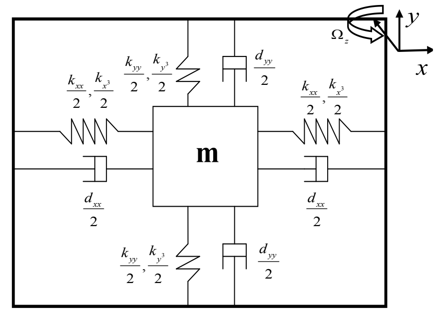

Fig.1. MEMS gyroscope with nonlinear effective spring

3. DESIGN OF DOUBLY-FED CONTROLLER

H-infinity control strategy using T-S model for MEMS gyroscopes is shown as in Fig.2. Based on the Lyapunov theory and the H infinite theory, LMI inequality and the doubly-fed control matrix K_f, K_b are designed. The controller consists of two parts: the feedforward controller U_f and the feedback controller U_b . The controller is implemented on the nonlinear model and T-S model at the same time in the presence of external disturbance to prove the accuracy of the T-S model and the feasibility of the proposed control scheme on the nonlinear model of MEMS gyroscope.

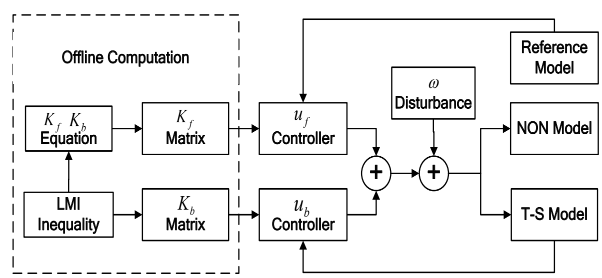

Fig.2. The block diagram of H infinity control

Fuzzy model addresses the imprecision of the input and output variables directly by defining them with fuzzy sets in the form of membership functions. T-S model is based on a set of fuzzy rules to describe a global nonlinear system in terms of a set of local linear models which are smoothly connected by fuzzy membership functions. T-S fuzzy models include two kinds of knowledge: one is qualitative knowledge represented by fuzzy IF-THEN rules, and the other is quantitative knowledge represented by local linear models. The MIMO T-S fuzzy model with input disturbance of the MEMS gyroscope is established based on the non-dimensional motion equation of MEMS gyroscope (3). A common linearization approach to construct local linear models is provided in [15]. The T-S fuzzy model of the MEMS gyroscope could be composed by 9 IF–THEN rules, which include both fuzzy inference rules and local analytic linear models. The ith rule has the form

Rule i : *IF* x *is about* M_{i1} *and* y *is about* M_{i2} *and* $\dot{x}$ *is about* M_{i3} *and* $\dot{y}$ *is about* M_{i4}

THEN $\dot{q} = A_i q + B_i u + H_i \omega \qquad i = 1, 2, \cdots, 9$